

# Corporate Governance and Managerial Reputational Concerns\*

Jonathan B. Cohn<sup>†</sup>      Uday Rajan<sup>‡</sup>

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<sup>†</sup>McCombs School of Business, University of Texas at Austin;

E-mail: [Jonathan.Cohn@mcombs.utexas.edu](mailto:Jonathan.Cohn@mcombs.utexas.edu).

<sup>‡</sup>Ross School of Business, University of Michigan, Ann Arbor; E-mail: [urajan@umich.edu](mailto:urajan@umich.edu).

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## Abstract

We consider the interaction of internal and external corporate governance when a corporate manager cares both about maximizing firm value and about his own reputation. External governance is represented by an outsider who generates information about the firm and becomes an activist shareholder. Due to reputational concerns, a manager with low skill is overly reluctant to reverse a decision about a project when faced with negative information. Internal governance, in the form of a board of directors, can improve the outcome by overturning the decision of the manager. However, in some cases, active internal governance exacerbates a manager's concerns about his reputation, making him more reluctant to voluntarily implement change. As a result, it may be optimal for a board to be passive rather than interventionist. The benefit to the board from an interventionist policy increases as the external signal becomes stronger. In the absence of external governance, the board optimally chooses to be passive as well. As the precision of the outsider's signal increases, internal and external governance are first substitutes, and then complements: the board invests more in internal governance as external governance improves. However, if the external signal is strong enough, the board simply free-rides on this signal. At this point, board behavior is either completely passive or extremely interventionist. Hence, the relationship between internal and external governance is non-monotone.

# 1 Introduction

Shareholder activism to force policy changes at firms has increased in recent years, and represents an important new dimension of the market for corporate control. Activist investors are often influential by persuading managers and boards to implement changes, without acquiring enough shares to gain direct control of the firm. For example, in 2006, Nelson Peltz succeeded in forcing Heinz to divest numerous brands added earlier in the tenure of the CEO, Bill Johnson, despite owing just 5.4% of the firm's equity. Not all investor activism campaigns are successful, as witnessed by Carl Icahn's unsuccessful attempt to force a makeover at Time Warner in 2006. As Bray, et al. (2008) document, activist hedge funds regularly cooperate with management and the board of directors, achieve some success about two-thirds of the time, and can have a significant impact on value when they attempt to change firm strategy.<sup>1</sup>

If a manager's interests are fully aligned with shareholders, he will simply incorporate the information of any activist investors and reach a value-maximizing decision. However, a manager also cares about his own reputation, which affects his future labor market outcomes. Reputational concerns induce stubbornness, and make a manager reluctant to reverse prior decisions.<sup>2</sup> Evidence of such reluctance can be seen in the high rate of divestitures following a change in management, and the subsequent improvement in firm value.<sup>3</sup> By continuing a range of inefficient projects for too long, a reputation-conscious manager can cause significant loss of value at a firm.

We study the roles of an activist investor and a board of directors in disciplining a manager who cares about his reputation. In our model, the board represents the primary internal governance mechanism at a firm. The activist investor provides external governance by generating information about the firm's projects. The manager's reluctance to change course may lead to a disagreement with the activist over the optimal policy at the firm. The firm's overall strategy is determined by the interaction among the manager, the activist, and the board.

We identify an important role for the board in arbitrating between a manager and an activist investor when they disagree about the firm's strategic direction. The board's optimal governance policy must take into account both the possible presence of the activist and the preferences of the manager. In some cases, optimal internal governance requires the board to blindly defer to the manager, even though active governance would ex post

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<sup>1</sup>Gillan and Starks (2007) document several additional sources of shareholder activism, as well as its increased incidence over the years.

<sup>2</sup>For example, a manager concerned about his reputation will not divest often enough (Boot, 1992), may fail to ignore sunk costs (Kanodia, Bushman and Dickhaut, 1989), and will be inflexible about altering investment plans over time (Prendergast and Stole, 1996).

<sup>3</sup>Weisbach (1995) examines divestitures following a CEO turnover, and Bhagat, Shleifer and Vishny (1992) following a hostile takeover.

improve firm value. A commitment to being passive improves managerial behavior *ex ante*. In other cases, the board must “overinvest” in governance to induce a potential activist to take action.

In our model, a manager must choose between two mutually-exclusive projects with uncertain payoffs. A project is interpreted as a decision about the broad strategic direction of the firm. The manager obtains a signal about the relative payoffs of the projects. The precision of his information is determined by his ability, which can be high or low. The firm also has a board of directors, which can gather information about the manager at a cost and can veto the manager’s decision. The board’s objective is to maximize firm value. The manager, on the other hand, cares both about the value of the firm and about his reputation (i.e., investors’ beliefs about his ability).

At the beginning of the game, the board may invest in a screening technology that (later in the game) produces a noisy signal of the manager’s ability. Then, having observed both his own type and a signal about project payoffs, the manager chooses one of the two projects. At the next stage, an activist investor (henceforth “activist” or “outsider”) chooses whether to generate additional information about the projects. If generated, the outsider’s information is made public, after which the manager has the option of switching projects. If the outsider’s signal agrees with the manager’s information, shareholder value is maximized by continuing with the initial project. However, if the signals disagree, the value-maximizing project depends on the manager’s ability: It is optimal to continue with a project chosen by a high-ability manager but switch to the other project if the manager’s ability is low. The board then receives its signal about the manager’s ability, and decides whether to intervene and overrule the manager’s decision.<sup>4</sup>

Boot (1992) considers a model in which an outside raider can engage in a hostile takeover of a firm with a stubborn manager and improve value via a divestiture. We build on his work by introducing a role for internal governance (implemented by the board) when outsiders and managers more broadly disagree about the value-maximizing strategy. We describe the model in detail in Section 2.

In Section 3, we analyze the continuation game that results if the outsider chooses to acquire her own information. If the manager’s signal and the outsider’s signal about relative project payoffs are in agreement, there is no conflict. The manager stays with the original project, and the board naturally allows this decision to stand. The more interesting case is the one in which the signals are in conflict. In this case, the manager can choose to switch projects (“concede”) or stay with the original project (“fight”).

We consider equilibria in which the high-type manager fights with probability one. Thus, if the manager concedes, he must be a low type, and the board maximizes value by allow-

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<sup>4</sup>Thus, in our model the board plays a similar role as in the model of Hermalin and Weisbach (1998), in which a board can generate a signal about a CEO and can fire and replace the CEO.

ing his reversal to stand. However, because the low-type manager is conscious about his reputation, he may choose to fight even when the alternative project has a higher expected payoff than the one he originally chose. If he fights, the board may either remain passive or intervene in project choice. In the latter case, it can further choose to overrule the manager only if it believes the manager has the low type, which we term “informed” governance, or in all cases (“sledgehammer” governance).

If the low-ability manager does not care too much about his reputation, he concedes when he anticipates informed governance. The result is a separating equilibrium that always implements the value-maximizing project. However, if he cares a lot about his reputation, the low-ability manager fights, resulting in a pooling equilibrium. When he is somewhat, but not overly, conscious about his reputation, a hybrid equilibrium obtains in which he mixes between fighting and conceding. Both the pooling and hybrid equilibria are inefficient, with the less valuable project being pursued at least some of the time.

One may expect that a more active board could mitigate this inefficiency. However, in the hybrid equilibrium, improved internal governance increases the stubbornness of the low-ability manager. The intuition for this key result is that, since the board only overrules the manager when it knows he has low ability, fighting and not being overruled sends investors a positive signal about the manager’s ability. The strength of this certification effect increases with the precision of the board’s signal. As a result, when the board invests more in its signal, the low-ability manager has a stronger incentive to fight.

The pooling and hybrid equilibria with informed governance only exist when external governance is relatively weak; i.e., the outsider’s signal is imprecise. With strong external governance, a pooling equilibrium with sledgehammer governance exists instead. The board essentially free rides off the outsider’s high-precision information. In doing so, it avoids the cost of generating information it would need to selectively overturn low ability managers. This equilibrium is also inefficient, since even the high-ability manager’s decision to fight is overruled.

We consider the optimal decision of the outsider in Section 4. The outsider incurs a fixed cost if she acquires information about the firm, and captures some of the resultant improvement in cash flow. She enters (i.e., acquires information about the firm) if the precision of her signal is sufficiently high, where the exact threshold depends both on the potential for agency conflict and on the equilibrium in the continuation game.

In Section 5, we consider the board’s investment in the screening technology at the start of the game. A manager who is not conscious about his reputation always chooses the value-maximizing project, so the board does not need to screen. Even when the manager is moderately conscious about his reputation, the board optimally invests nothing in acquiring information about him. In this case, the hybrid equilibrium obtains, resulting in the low-ability manager fighting with positive probability. As mentioned earlier, better screening

by the board would lead the low-ability manager to fight more often, exactly offsetting the benefit of the more precise signal. Since investing in the signal is costly, the board optimally chooses to not screen at all. Hence, it allows the manager's decision on the project to stand. That is, the board simply defers to the manager, even though the low-ability manager will too often choose to continue his original project.

When the manager cares enough about his reputation that he always fights, the optimal screening level for the board depends on the strength of external governance (i.e., the precision of the outsider's signal). The activist's payoff increases when the board is more likely to overturn the low-ability manager. If the board implements informed governance, the latter probability in turn depends on the precision of the board's signal. Thus, by over-investing in screening (relative to the value-maximizing level when the outsider's presence is taken for granted), the board can induce the outsider to be active. If the outsider's information is very imprecise, such an investment is too costly. As a result, there is no governance in equilibrium. However, in an intermediate range of precision, the board over-invests in internal governance and does induce the activist to enter. As the quality of the activist's signal improves, the board reduces its over-investment, so that improved external governance is accompanied by less intense internal governance.

Once the outsider's signal is sufficiently precise, the board no longer needs to overinvest in its own signal to induce the outsider to enter. The board then chooses a screening level that optimally trades off the cash flow benefit from intervention with the direct cost of its signal. The optimal level increases as the precision of the outsider's signal increases, since it is now more valuable to overturn the low-ability manager. Thus, over this parameter range, external and internal governance are complements. Finally, when the outsider's signal is very precise, the board simply free-rides on the outsider's information, resulting in sledgehammer governance. The overall relationship between external and internal governance is thus complex and non-monotone.

Our paper falls in the strand of the corporate governance literature that examines optimal allocation of control and decision-making within the firm. Bebchuk (2005) concludes that firm value would be improved by a greater concentration of power in the hands of shareholders (or their representatives on the board). Our work is more in the spirit of Harris and Raviv (2008b), who show that activist shareholders should not always have control over corporate decisions. As in their framework, an activist shareholder in our model is only partially informed. While the board retains ultimate authority in our model, the equilibria in which it is completely passive may be interpreted as situations in which it cedes control to the manager. Burkart, Gromb and Panunzi (1997) and Almazan and Suarez (2003) argue that ex post transfer of control to shareholders or boards changes the nature of the agency conflict between management and shareholders. However, unlike in their papers, we find that it can either worsen or improve the agency conflict.

Internal and external governance are likely to function as substitutes if they perform the same function of disciplining managers (see, for example, Fama, 1980, Fama and Jensen, 1983, and Williamson, 1983). Acharya, Myers and Rajan (2008), on the other hand, suggest that external governance (by the board) complements internal governance (by subordinates within the firm). Immordino and Pagano (2009) also consider the interaction of internal governance (i.e., actions by a board) with external governance (in their case, the actions of an outside auditor), and find the two can be complementary under some conditions. Our model predicts a non-monotonic relationship between internal and external governance. When external governance is weak, it either has no relationship to or is complementary to internal governance. However, when external governance is strong, it is a substitute for internal governance: the board chooses to free-ride on the information of the outsider, and invests nothing in internal governance.<sup>5</sup>

As a final point, Gompers, Ishii and Metrick (2003) and Bebchuk, Cohen, and Ferrell (2004) have constructed widely-used empirical indices of corporate governance that lump together both internal and external governance measures. However, our results suggest that the interplay of internal and external governance can be quite complex, and simple aggregation may not be a reliable way to measure the expected effectiveness of governance. For example, in our model, there is sometimes a negative relationship between firm value and internal governance: the board can improve firm value by committing to be less active.

## 2 Model

A publicly-traded firm faces a choice between two mutually exclusive projects. Each project yields a cash flow of either 0 or 1 at time 4. There are two possible future states. In state  $x_A$ , project  $A$  yields a cash flow of 1 and project  $B$  earns 0. In state  $x_B$ , project  $A$  earns 0 and project  $B$  earns 1. The ex ante probability of state  $x_A$  is  $\frac{1}{2}$ . The firm is operated by a manager who has a type  $\theta \in \{\theta_H, \theta_L\}$ . The manager receives an informative signal about the project, with the high type having a signal of greater precision.

There are two stages at time 0. First, the board of directors of the firm invests in an internal governance mechanism that provides information about the type of the manager. The amount that the board invests in this mechanism is observed by the manager. This mechanism can be interpreted as a set of regular reports that the manager is compelled

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<sup>5</sup>Empirical research on the relationship between internal and external governance has yielded mixed results. For example, Mayers, Shivdasani and Smith (1997) find that mutual insurance companies, which are not readily taken over, have more outside directors than stock insurance companies, suggesting that internal governance is a substitute for external governance. Brickley and James (1987), on the other hand, find that banks in states that prohibited bank takeovers tended to have fewer outside directors than those in states without such takeover restrictions, suggesting that internal governance complements external governance.

to supply, but could also incorporate soft information about the manager's ability that the board gathers from conversations with the manager, other officers, and experts in corporate management practices.

The signal produced by the internal governance mechanism takes some time to generate, and therefore the board must choose how much to invest in it at time 0; that is, the board cannot wait until later periods to choose the amount that it invests. The signal is binary, with  $s_B \in \{H, L\}$ . We assume that  $\text{Prob}(s_B = H \mid \theta_H) = 1$  (so the high-ability manager generates signal  $L$  with probability 0) and  $\text{Prob}(s_B = H \mid \theta_L) = 1 - \alpha$  (so the low-ability manager generates signal  $L$  with probability  $\alpha$ ). Thus, when  $\alpha = 0$ , the board signal is completely uninformative (since both manager types generate the signal  $H$  with probability 1), and the signal becomes fully informative as  $\alpha$  approaches 1. The internal governance mechanism is parameterized by the precision of the signal ( $\alpha$ ), which is chosen by the board. A signal of precision  $\alpha$  is obtained at a cost  $c(\alpha)$ . The cost function is strictly increasing and strictly convex in  $\alpha$ . In addition, we assume that  $c(0) = 0$ ,  $c'(0) = 0$  and  $\lim_{\alpha \rightarrow 1} c'(\alpha) = \infty$ . The restrictions ensure that the board will choose a level of  $\alpha$  strictly less than 1.

At time 0, after the board has chosen  $\alpha$ , the manager receives a signal about the true state  $s_M \in \{A, B\}$ , and embarks on a project. The informativeness of the manager's signal depends on the ability of the manager,  $\theta$ . Specifically,  $\text{Prob}(s_M = k \mid X = x_k) = \theta$ . The ability of the manager represents his type. The manager may have either high ( $\theta_H$ ) or low ( $\theta_L$ ) ability, with  $1 \geq \theta_H > \theta_L \geq \frac{1}{2}$ . The unconditional probability that the manager is type  $\theta_H$  is denoted by  $q \in (0, 1)$ . The manager's payoff, described in detail below, depends both on the cash flow from the project and on investors' posterior beliefs about the manager's type. The manager knows his own type, but other parties in the model do not.<sup>6</sup> Having observed his own signal, the manager begins either project  $A$  or project  $B$ . It is a best response for the manager to choose project  $k$  if his signal is  $k$ .

At time 1, an outsider chooses whether to generate a signal about the true state,  $s_E \in \{A, B\}$ , or to stay out of the game. The outsider may be thought of as an external activist investor, who acquires her own signal about the optimal project. Alternatively, the outsider may be an existing shareholder who wishes to force a policy change at the firm, and is external to the current power structure at the firm.

The outsider's signal is less precise than the signal of a high-ability manager, but more precise than the signal of a low-ability manager. In particular,  $\text{Prob}(s_E = k \mid X = x_k) = \psi$ , where  $\theta_L < \psi < \theta_H$ . Thus, if the manager's and outsider's signals disagree, the efficient outcome accords with the manager's signal if the manager has high ability, but with the outsider's signal if the manager has low ability.

Suppose the outsider does generate a signal at time 1. This signal is assumed to be

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<sup>6</sup>Here, we follow Boot (1992) and Prendergast and Stole (1996), rather than the career concerns model of Holmström (1999), in which an agent does not know his own type.

publicly observed. Then, at time 2, the manager has the opportunity to switch projects at a minimal cost.<sup>7</sup> After the manager has made his choice, the board obtains its signal about the manager's ability, and decides whether to uphold the manager's decision or implement the alternative project. It is optimal for the board to let the manager proceed with his chosen project if either the outsider does not generate a signal or the manager chooses the project favored by the outsider's signal. However, if the manager chooses the project that conflicts with the outsider's signal, the board may overturn his choice.

At time 3, investors form posterior beliefs about the type of the manager. Let  $\mu$  denote the posterior probability at time 1 that the manager has type  $\theta_H$ . This posterior probability depends on the strategies of the manager and the board, on the outsider's signal, and on the observed actions of the manager and the board.

Finally, at time 4, the cash flow from the project is realized as either 0 or 1. The project is therefore a long-term project, whose outcome is not known in the short-run. However, the manager's labor market opportunities depend on investors' short-run beliefs over his ability.<sup>8</sup> Figure 1 displays the sequence of events in the model.

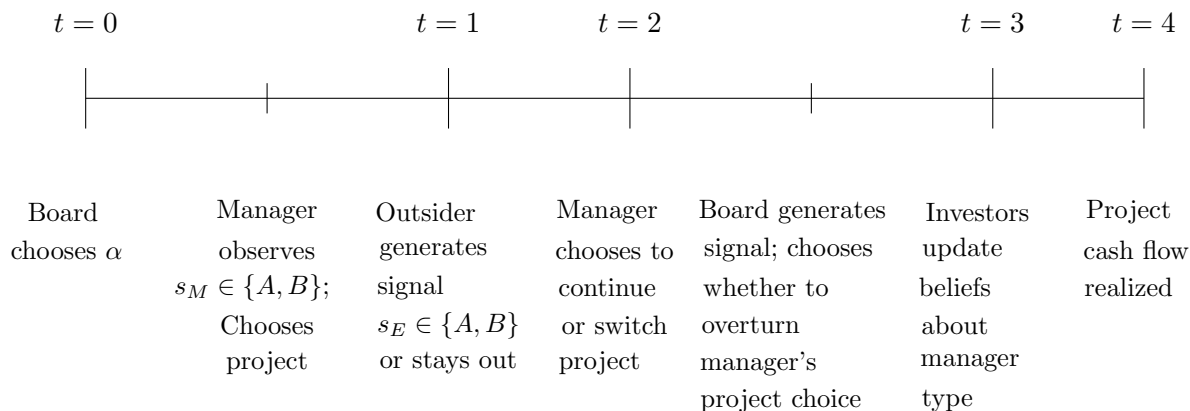


Figure 1: Sequence of events

Let  $v$  be the value of the firm at time 4; that is,  $v$  is the cash flow of the project minus the cost of the board's signal,  $c(\alpha)$ . Further, let  $\theta^\mu = \mu\theta_H + (1 - \mu)\theta_L$  be the investors' posterior expectation (at time 3) of manager type. The manager's payoff is then

$$U_M = \beta v + (1 - \beta)\theta^\mu,$$

where  $\beta \in (0, 1)$ . Thus, the manager cares both about the success of the project and about

<sup>7</sup>The cost ensures that the manager strictly prefers to choose the project at stage 1 that conforms to his own signal. This makes the initial project choice informative about the manager's signal.

<sup>8</sup>For example, if the manager were to leave the firm at time 1, his compensation in the new job would depend on his perceived ability (see, e.g., Harris and Holmström, 1982).

his reputation, i.e., investors' beliefs about his type.<sup>9</sup>

The board represents the shareholders, who care only about the overall value of the firm (that is, expected project cash flow less any resources spent on acquiring a signal about the manager). Thus, the board's payoff function is just  $U_B = v$ . We defer a discussion of the outsider's payoff to Section 4. All parties are risk-neutral, and so maximize their respective expected payoffs.

We interpret  $\psi$ , the precision of the public signal, as a proxy for the strength of external corporate governance. The public signal represents factors outside the control of the board or the firm that nevertheless affect the manager's behavior. Although the signal itself does not have a direct governance component in the sense of requiring the manager to undertake a particular action, it plays two roles in the governance process. First, it influences the manager's choice of action, since the manager cares about the payoff on the project. It also helps refine investors' beliefs over manager type: As we show below, a type  $\theta_L$  manager is more likely to be confronted by a public signal that conflicts with his own.

The board plays two roles in the governance process. First, at time 0, it chooses an optimal level of screening by deciding how much to invest in the internal governance mechanism, which in turn affects the manager's action at time 2. Second, at time 2, it decides whether to directly intervene in the operations of the firm and implement a project contrary to the manager's choice.

We consider a perfect Bayesian equilibrium of the game. Therefore, the board cannot commit to its overturning strategy at time 2. Instead, its action must be a best response given its own choice of  $\alpha$  at time 0 and given the strategy of the manager. Further, the beliefs of the board at time 2 and investors at time 3 about the type of the manager must be consistent with Bayes' rule whenever possible.

We focus on equilibria in which, at time 0, the manager chooses the project that is favored by his signal. Thus, if  $s_M = A$ , project  $A$  is chosen, and if  $s_M = B$ , project  $B$  is chosen. At time 2, if  $s_E = s_M$  or if the outsider chooses to stay out, the manager has no reason to switch to the other project, and will continue with the project he had chosen earlier. In this case, there is no reason for the board to intervene at time 2.

Thus, the continuation game at time 2 is relevant only if the outsider enters and  $s_E \neq s_M$  (that is, the manager and outsider receive conflicting signals). Under this scenario, the manager must decide whether to continue with the current project, or switch to the other project. Since the game is symmetric in projects  $A$  and  $B$ , the decision is similar regardless of which project was adopted at stage 1. We restrict attention to equilibria in which the continuation probability is invariant to the project chosen at time 0, and hence to the actual realization of  $s_M$ . Let  $\sigma_k$ , for  $k \in \{L, H\}$ , denote the probability the manager continues

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<sup>9</sup>As in Prendergast and Stole (1996), the manager's payoff depends directly on the market's expectation of his ability.

with the current project at time 2, when the manager's type is  $k$  and  $s_E \neq s_M$ . Such a continuation puts the manager in direct conflict with the outsider, and we refer to this choice of strategy as "Fight". If the manager instead adopts the project favored by the public signal, we refer to his action as "Concede."

If  $s_E \neq s_M$ , the board must decide whether to overturn the manager's choice of project. Again, given the symmetry of the game, we consider only equilibria in which the board's actions do not depend on either the project chosen at time 0, or the outsider's signal at time 1. We also restrict attention to equilibria in which, if the manager concedes, the board allows his decision to stand. In the equilibria we consider in Section 3, the high-type fights with probability 1, rendering this assumption innocuous.

Suppose the signals of the manager and outsider disagree, and the manager fights. In any perfect Bayesian equilibrium, the board must overturn the manager's decision whenever it knows the manager has the low type (i.e., the board obtains signal  $L$ ). Let  $\gamma$  denote the probability the board overrules the manager when the manager fights and the board's signal is  $H$ . Finally, let  $\xi$  denote the outsider's optimal decision at time 0, with  $\xi = 0$  implying that the outsider stays out (i.e., does not acquire information about the firm) and  $\xi = 1$  that the outsider enters (i.e., generates a signal).

Let  $\sigma = (\sigma_H, \sigma_L)$ . With a slight abuse of terminology, we describe an equilibrium only in terms of  $(\alpha, \xi, \sigma, \gamma)$ , with beliefs for the board at time 2 and investors at time 3 that are consistent with Bayes' rule, wherever possible.

### 3 Optimal Strategies of Manager and Board at Time 2

We begin by considering the continuation game starting at time 2. The board has chosen  $\alpha$  at  $t = 0$ ; for now we hold this choice of  $\alpha$  fixed. Since the board will never choose  $\alpha = 1$ , we fix  $\alpha$  to be strictly less than 1. If the outsider stays out at time 1, it is optimal for the board to allow the manager to proceed with his chosen project (since  $\theta_L \geq \frac{1}{2}$ ). Hence, in this section, we focus on the case where the outsider enters at time 1, and  $s_E \neq s_M$ .

We consider equilibria that are symmetric in the initial choice of project. Hence, in the analysis of the continuation game, we assume without loss of generality that the manager observes signal  $A$  at stage 1. A conflict occurs only if the outsider obtains signal  $B$ . We focus on this case.

Since  $s_M = A$ , the manager chooses project  $A$  at  $t = 0$ . Let  $\lambda_i$  be the probability that the signals of the manager and the outsider disagree when the manager has type  $\theta_i$ . Then,  $\lambda_i = \theta_i(1 - \psi) + \psi(1 - \theta_i)$ , for  $i = H, L$ . Define  $\delta_i$  as the probability that  $x_A = 1$  if  $s_M = A$  and  $s_E = B$ , when  $\theta = \theta_i$ . Then,  $\delta_i = \frac{\theta_i(1-\psi)}{\lambda_i}$  for  $i = L, H$ , with  $\delta_L < \frac{1}{2} < \delta_H$ . Given that the type of the manager is  $\theta_i$ , the manager received signal  $s_M = A$ , and the outsider's signal is  $s_E = B$ , the expected cash flow from project  $A$  is  $\delta_i$  and that from project  $B$  is

$1 - \delta_i$ .

Suppose the type  $\theta_i$  manager fights with probability  $\sigma_i$ , and the board overturns the manager on receiving signal  $H$  with probability  $\gamma$ . Let  $\mu_f(\alpha)$  be the posterior probability that the manager has type  $H$ , given that the manager fights and the board receives signal  $H$ . Further, let  $\mu_c(\alpha)$  be the posterior probability that the manager has high type, given that the manager concedes and the board receives signal  $H$ .<sup>10</sup> These posterior beliefs are constructed as follows.

The posterior probability the manager has type  $H$  given that  $s_M = A$  and  $s_E = B$  is  $\frac{q\lambda_H}{q\lambda_H+(1-q)\lambda_L}$ . Then, whenever the respective denominators are positive,

$$\mu_f(\alpha) = \frac{q\lambda_H\sigma_H}{q\lambda_H\sigma_H + (1-\alpha)(1-q)\lambda_L\sigma_L}, \quad (1)$$

$$\mu_c(\alpha) = \frac{q\lambda_H(1-\sigma_H)}{q\lambda_H(1-\sigma_H) + (1-\alpha)(1-q)\lambda_L(1-\sigma_L)}. \quad (2)$$

We first characterize the best responses of the board and each type of manager in the continuation game at time 2. Recall that if the board receives signal  $L$ , it knows the manager has the low type, and so will overturn the manager if  $s_E = B$  and he fights. If it obtains signal  $H$ , it will overturn the manager if the posterior probability that he has the high type is sufficiently low.

**Lemma 1.** *At time 2, the best responses in the continuation game are as follows:*

- (i) *The board sets  $\gamma = 1$  if  $\mu_f(\alpha) < \frac{1-2\delta_L}{2(\delta_H-\delta_L)}$ ,  $\gamma = 0$  if  $\mu_f(\alpha) > \frac{1-2\delta_L}{2(\delta_H-\delta_L)}$ , and chooses any  $\gamma \in [0, 1]$  if  $\mu_f(\alpha) = \frac{1-2\delta_L}{2(\delta_H-\delta_L)}$ .*
- (ii) *For  $i = H, L$ , the type  $i$  manager sets  $\sigma_i = 1$  if  $\mu_f(\alpha) > \mu_c(\alpha) + (1-\gamma)\frac{\beta}{1-\beta}\frac{1-2\delta_i}{\theta_H-\theta_L}$ ,  $\sigma_i = 0$  if  $\mu_f(\alpha) < \mu_c(\alpha) + (1-\gamma)\frac{\beta}{1-\beta}\frac{1-2\delta_i}{\theta_H-\theta_L}$  and chooses any  $\sigma_i \in [0, 1]$  if  $\mu_f(\alpha) = \mu_c(\alpha) + (1-\gamma)\frac{\beta}{1-\beta}\frac{1-2\delta_i}{\theta_H-\theta_L}$ .*

In the manager's best response condition, the term  $\mu_c(\alpha) + (1-\gamma)\frac{\beta}{1-\beta}\frac{1-2\delta_i}{\theta_H-\theta_L}$  represents a threshold belief. If the investors' posterior belief that the manager has type  $\theta_H$  exceeds this threshold, the manager fights. Otherwise, he concedes. Recall that  $\delta_H > \frac{1}{2} > \delta_L$ . Suppose  $\gamma < 1$ , so that there is positive probability the manager will be allowed to implement the project he has chosen. Then, the threshold belief for the high type is strictly lower than the corresponding threshold for the low type. Further, the high-type manager will fight even when  $\mu_f(\alpha) < \mu_c(\alpha)$ . If he concedes, project  $B$  is implemented. This project has expected

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<sup>10</sup>For now, we assume that, even if the manager concedes, the board observes a signal about his type. This signal is revealed to investors, who update their beliefs accordingly.

cash flow  $1 - \delta_H$ . If he fights, as long as project  $A$  is implemented with positive probability, there is an improvement in expected cash flow. Converse reasoning applies to the low-type manager. Conceding may lead to project  $A$  (in this case, the inefficient project) being implemented with positive probability. Thus, the low-type manager must strictly gain on the reputational component of payoff to make fighting worthwhile.

Next, we show that equilibria in the continuation game can be characterized as follows. If the board overturns the manager with probability less than 1 when it obtains signal  $H$  (i.e., if  $\gamma < 1$ ), then it must be that either the high-type manager fights with probability one, or both types of manager fight with probability zero. If, instead, the board always overturns the manager when it receives the high signal, both types of manager must fight with equal probability.

**Lemma 2.** *Consider an equilibrium of the continuation game at time 2.*

- (i) *If  $\gamma < 1$ , either  $\sigma_H = 1$  or  $\sigma_H = \sigma_L = 0$ .*
- (ii) *If  $\gamma = 1$ ,  $\sigma_H = \sigma_L$ .*

Consider any equilibrium of the continuation game in which both types of manager concede with probability one. Such an equilibrium is sustained by an off-equilibrium belief that there is a sufficiently large probability a manager who fights has the low type. Now, suppose the high-type manager deviates. If, following the deviation, the board overrules the manager with probability less than 1, the expected cash flow of the firm is strictly greater. Conversely, if the low-type manager were to deviate and the board responds with  $\gamma < 1$ , the expected cash flow of the firm strictly falls. Hence, if  $\beta$  is sufficiently high, only the high-type manager has an incentive to deviate. In the spirit of the Cho and Kreps (1985) Intuitive Criterion, following a deviation the board should believe it is dealing with the high type, breaking the equilibrium.

Going forward, in considering equilibria in which  $\gamma < 1$ , we focus on the case  $\sigma_H = 1$ ; that is, the high-type manager fights. In some of the equilibria we consider, the low-type manager concedes with positive probability. When the low-type manager also fights with probability one, the equilibrium can be sustained by the off-equilibrium belief that a concession comes from the low type. Thus, following a concession, it is optimal for the board to allow the manager to proceed with his ultimate choice of project.

We also restrict attention to equilibria in which the board plays a pure strategy. That is, the board either sets  $\gamma = 0$  or  $\gamma = 1$ . Recall that the board always overrules the manager if it obtains the low signal. If  $\gamma = 0$ , the board upholds the manager on obtaining the high signal. That is, the board partially screens the manager type, and hence displays

what we call “informed” governance. Conversely, if  $\gamma = 1$ , the board overrules the manager regardless of the signal it obtains. In such cases, we say the board exhibits “sledgehammer” governance.

We first consider equilibria in which the high-type manager fights when his signal disagrees with the public signal and the board exhibits informed governance. In such an equilibrium, the low-type manager faces a tradeoff between fighting and conceding. If he fights, then, with probability  $\alpha$ , the board identifies him as a low type and overrules him. Thus, with probability  $1 - \alpha$ , project  $A$  is continued. The low type finds this costly because the expected payoff from project  $A$ ,  $\delta_L$ , is less than the expected payoff from project  $B$ ,  $1 - \delta_L$ . However, fighting allows him to pool with the high type with probability  $1 - \alpha$ , which confers a reputational benefit.

If the low type concedes, the firm implements project  $B$  and investors learn that the manager is a low type (since the high type never concedes). The low type then obtains a payoff  $\beta(1 - \delta_L) + (1 - \beta)\theta_L$ . He receives exactly the same payoff if he fights and is overruled by the board. The low-type manager is therefore indifferent between these two outcomes.<sup>11</sup> Thus he fights if and only if his payoff from fighting and *not* being overruled exceeds his payoff from conceding. In this scenario, the firm implements the wrong project. However, the low-type manager obtains a reputational benefit, since investors’ posterior expectation about his type must exceed  $\theta_L$ . Therefore, he concedes only if  $\beta$  (the extent to which he cares about firm value) is high enough to outweigh the reputational benefit from fighting. Specifically, define

$$\beta_s(\psi) = \frac{1}{1 + \frac{1 - 2\delta_L}{\theta_H - \theta_L}}. \quad (3)$$

Note that  $\beta_s$  declines in  $\psi$  (since  $\delta_L$  decreases when  $\psi$  increases), but is independent of  $\alpha$ , the precision of the board’s signal.

**Proposition 1.** *If (and only if)  $\beta \geq \beta_s(\psi)$ , there exists a separating equilibrium in the continuation game at time 2 that induces efficient project selection. In this equilibrium,  $\sigma_H = 1$ ,  $\sigma_L = 0$  and  $\gamma = 0$ .*

When  $\beta$  is high, manager and shareholder interests are well-aligned. Therefore, the manager responds to the arrival of the outsider’s signal by choosing the project with the highest expected payoff. On the other hand, if  $\beta$  is below the threshold value  $\beta_s$ , any

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<sup>11</sup>One could imagine a cost to the manager of fighting and being overruled, compared to conceding quietly. A proportional cost strengthens rather than weakens our results. A fixed cost changes the details of the equilibria, but not the qualitative features. Since such a cost complicates the analysis without adding insight, we ignore the possibility.

continuation equilibrium will be characterized by some degree of pooling and hence of inefficiency in terms of project choice.

If  $\beta$  is very low, the manager focuses primarily on his reputation and places little weight on firm value. In this case, the low-type manager would like to pool with the high-type manager by fighting. However, such pooling results in the frequent implementation of inefficient projects, unless the board intervenes. As a result, the board may find it optimal to increase  $\gamma$  when it expects the low-type manager to fight. Its decision to intervene depends on the precision of the public signal.

When the public signal is relatively precise, disagreements with the manager's signal are more likely to occur when the manager has a low type. Given such a disagreement, the expected payoff of project  $B$  increases with the precision of the public signal, while that of project  $A$  falls. Both these factors imply that the benefit to the board of overruling the manager increases with  $\psi$ . In fact, if  $\psi$  is sufficiently high, the board is willing to overrule the manager even when it obtains signal  $H$ . Therefore, a necessary condition for a pooling equilibrium with informed governance is that  $\psi$  is sufficiently low. Specifically, let

$$\psi_f(\alpha) = \frac{q\theta_H + (1-\alpha)(1-q)\theta_L}{q + (1-\alpha)(1-q)} \quad (4)$$

be the conditional expectation of  $\theta$ , given that both types fight and the board obtains a high signal. It is straightforward to show that  $\psi_f(\alpha)$  increases in  $\alpha$ . The board implements informed governance (i.e., sets  $\gamma = 0$ ) only if  $\psi \leq \psi_f(\alpha)$ .

Of course, for the low-type manager to fight, it must be that  $\beta$  is low. Define

$$\beta_\ell(\alpha, \psi) = \frac{1}{1 + \frac{1-2\delta_L}{\theta_H - \theta_L} \left[ 1 + \frac{(1-\alpha)(1-q)\lambda_L}{q\lambda_H} \right]}. \quad (5)$$

Since  $\alpha < 1$ , it follows that  $\beta_\ell(\alpha, \psi) < \beta_s(\psi)$ . Further, notice that  $\beta_\ell(\alpha, \psi)$  increases in  $\alpha$ . A pooling equilibrium with informed governance exists when  $\beta \leq \beta_\ell(\alpha, \psi)$  and  $\psi \leq \psi_f(\alpha)$ .

**Proposition 2.** *A pooling equilibrium with informed governance exists in the continuation game at time 2 if and only if  $\beta \leq \beta_\ell(\alpha, \psi)$  and  $\psi \leq \psi_f(\alpha)$ . In such an equilibrium, both types of manager fight and the board overrules the manager only if it obtains the low signal. That is,  $\sigma_H = \sigma_L = 1$  and  $\gamma = 0$ .*

When  $\beta$  is in an intermediate range, the manager is somewhat, but not overly, conscious about his reputation. In this case, there can exist a hybrid equilibrium with informed governance, in which the high-ability manager fights and the low-ability manager mixes between fighting and conceding. The board allows the manager's project to continue if

it receives signal  $H$ , and overturns the manager only if it receives signal  $L$ . As with the pooling equilibrium in Proposition 2, for such a hybrid equilibrium to exist, the board must find it optimal to not overrule the manager when it obtains signal  $H$ . Define

$$\beta_b(\psi) = \frac{1}{1 + \frac{2(\delta_H - \delta_L)}{\theta_H - \theta_L}} \quad (6)$$

Since  $\delta_H > 1/2$ , it follows that for each value of  $\psi$ ,  $\beta_b < \beta_s$ . As with  $\beta_s$ ,  $\beta_b$  does not depend on  $\alpha$ .

Suppose that investors believe the low-type manager concedes with probability one, and the board allows the manager's decision to stand when it obtains signal  $H$ . Then, on observing that the board lets the manager proceed with his choice of project, investors believe he has the high type. This provides the low-type manager an incentive to fight, since the reputational component of his payoff improves. If  $\beta < \beta_s$ , the low-type manager does not care enough about firm value for the separating equilibrium in Proposition 1 to obtain. Hence, he does not concede with probability one.

Next, suppose that investors believe the low-type manager fights with probability one, and the board allows the manager to proceed with his project if it obtains signal  $H$ . In this case, investors' posterior expectation about type when the board allows the manager to proceed is lower than  $\theta_H$ . Thus, the reputational benefit of fighting is smaller than in the previous case. Therefore, if  $\beta$  is sufficiently high (but lower than  $\beta_s$ ), the low-type prefers to concede, breaking the pooling equilibrium in Proposition 2.

In the hybrid equilibrium, the low-type manager is indifferent between fighting and conceding. The probability that he fights,  $\sigma_L$ , depends on the parameters  $\beta$ ,  $\psi$ , and  $\alpha$ . In particular, we show that it increases with  $\alpha$ , the precision of the board's signal.

**Proposition 3.** *(i) A hybrid equilibrium with informed governance exists in the continuation game at time 2 if and only if  $\beta \in (\max\{\beta_\ell(\alpha, \psi), \beta_b(\psi)\}, \beta_s(\psi))$ . In such an equilibrium, the high-type manager fights, the low-type manager mixes between fighting and conceding, and the board overrules the manager only if it obtains the low signal. That is,  $\sigma_H = 1, \sigma_L \in (0, 1)$  and  $\gamma = 0$ .*

*(ii) In a hybrid equilibrium of the continuation game at time 2, the probability that the low-type manager fights increases with the precision of the board's signal about manager ability. That is,  $\frac{\partial \sigma_L}{\partial \alpha} > 0$ .*

Part (ii) of Proposition 3 establishes a crucial insight of this paper: In the hybrid equilibrium, stronger internal governance in the form of better screening by the board leads the low-ability manager to fight *more* often. In other words, better internal governance

exacerbates the agency conflict faced by the shareholders.

A higher value of  $\alpha$  implies that the low-type manager is more likely to be overturned if he fights. However, note that if he fights and is overruled, he obtains the exact same payoff (both in terms of firm value and on the reputational component) as he does on conceding. If he fights and is allowed to proceed with his choice of project, the effect on his payoff is more complicated. The inefficient project is implemented, which is costly. Compensating for this cost, the reputational component of his payoff is higher when  $\alpha$  increases. Essentially, being allowed to proceed by the board provides a noisy certification about his ability. Therefore, holding  $\sigma_L$  fixed, the low-type manager's payoff from fighting *increases* with  $\alpha$ . In turn, this results in an increase in  $\sigma_L$ , which reduces the reputational benefit of fighting and not being overruled so that, in equilibrium, the expected payoff from fighting and conceding are again equalized.

Next, we consider the case of an interventionist board. The board's own signal is noisy. Therefore, if the public signal is sufficiently precise and the board believes that the low-type manager fights often enough, it may be optimal for the board to overturn the manager even when it obtains signal  $H$ . Of course, the board always overturns the manager on obtaining signal  $L$ . Thus, in such an equilibrium, the board's action is independent of its own signal. An immediate implication is that knowing a manager was overturned has no information content for investors. Further, if a manager is always overturned by the board, both types are indifferent between fighting and conceding. Thus, in a continuation equilibrium with sledgehammer governance, the high type also may fight with probability less than one.

**Proposition 4.** *An equilibrium with sledgehammer governance exists in the continuation game at time 2 if and only if  $\psi \geq \psi_f(\alpha)$ . In such an equilibrium,  $\sigma_H = \sigma_L \in (0, 1]$  and  $\gamma = 1$ .*

Equilibria in which both types of manager mix between conceding and fighting cannot be dismissed by a refinement of beliefs, since there is no unreached information set. However, note that for both types of manager to mix, the expected cash flow of the firm must be the same regardless of whether the manager concedes or fights. Hence, imposing a selection on this class of equilibria does not affect the expected cash flow of the firm, and so does not affect the optimal action of the board. Therefore, when considering equilibria with  $\gamma = 1$ , without loss of generality we focus on the case that  $\sigma_H = \sigma_L = 1$ ; that is, both types of manager fight with probability one.

Now, suppose that  $\psi > \psi_f(\alpha)$  and  $\beta > \beta_b$ . Then, there are multiple equilibria in the continuation game. From Proposition 4, an equilibrium with sledgehammer governance exists when  $\psi > \psi_f(\alpha)$ , regardless of the value of  $\beta$ . However, Proposition 1 shows that if

$\beta \geq \beta_s$ , there is also a separating equilibrium. Further, from Proposition 3, if  $\beta \in (\beta_b, \beta_s)$ , there is also a hybrid equilibrium with informed governance. Whenever there are multiple equilibria in the continuation game for a fixed value of  $\alpha$ , we select the equilibrium that maximizes the expected payoff of the board, conditional on  $s_E \neq s_M$ . We show that the board prefers the separating or hybrid equilibrium to the pooling equilibrium with sledgehammer governance.

**Lemma 3.** *Suppose  $\psi \geq \psi_f(\alpha)$ . Then, if  $s_E \neq s_M$  and the manager fights:*

- (i) *If  $\beta \geq \beta_s(\psi)$ , the board's expected payoff is higher under a separating equilibrium than under the equilibrium with sledgehammer governance.*
- (ii)  *$\beta \in [\beta_b(\psi), \beta_s(\psi))$ , the board's expected payoff is higher under a hybrid equilibrium than under the equilibrium with sledgehammer governance.*

Therefore, if  $\beta \geq \beta_s(\psi)$ , we assume the separating equilibrium is played in the continuation game at time 2, regardless of the value of  $\psi$ . In the proof of Proposition 3, we show that  $\beta_b(\psi) > \beta_\ell(\alpha, \psi)$  for  $\psi > \psi_f(\alpha)$ . Thus, if  $\psi > \psi_f(\alpha)$  and  $\beta \in [\beta_b(\psi), \beta_s(\psi))$ , we fix the equilibrium in the continuation game to be the hybrid equilibrium.

In Figure 2, we illustrate the equilibria we consider at time 2, for different values of  $\beta$  and  $\psi$ . The parameters for this figure are set to  $\theta_H = 0.9$ ,  $\theta_L = 0.55$ ,  $q = 0.4$ , and  $\alpha = 0.5$ .

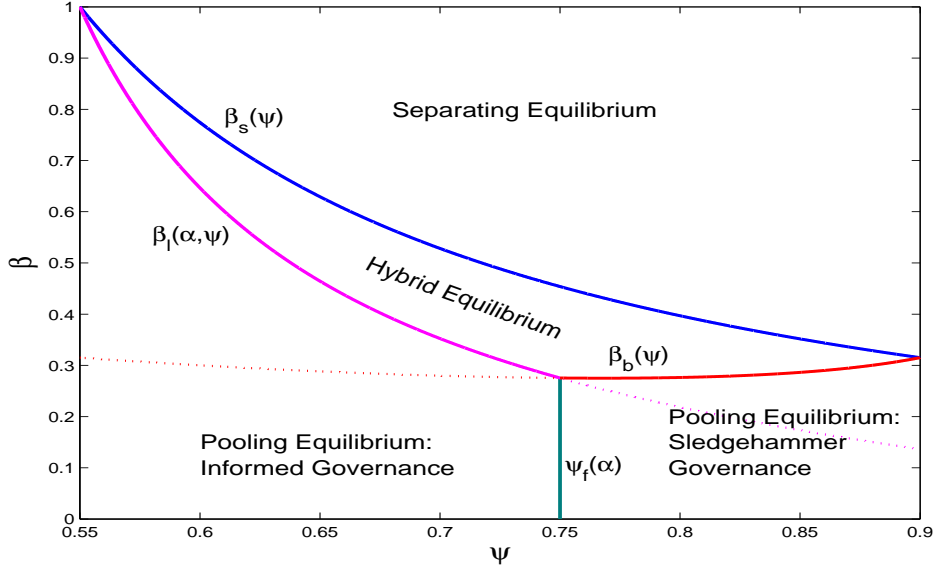
## 4 Optimal Decision of Outsider at Time 1

We now step back to time 1, and consider the optimal decision of the outsider. The board has chosen  $\alpha$  at time 0, and the outsider anticipates that, if she intervenes and generates a contrary signal, a continuation equilibrium  $(\sigma, \gamma)$  will be played at time 2.

We assume that the outsider can acquire a fraction  $\eta$  of the shares in the firm before time 2, that is, before she acquires information about the firm. For convenience, we further assume that the market values these shares at the expected value of the firm assuming the outsider will not generate a signal.<sup>12</sup> If the outsider chooses to stay out, she does not acquire a stake in the firm. In this case, the board does not intervene (since  $\theta_L \geq \frac{1}{2}$ , it is optimal to leave the manager alone even if the board finds out he has the low type). Let  $F_0 = q\theta_H + (1 - q)\theta_L$  denote the expected cash flow from the project in this case. The value

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<sup>12</sup>If investors could completely predict the presence of the outsider, the usual information acquisition problem arises: an agent will not acquire costly information if it is already incorporated into the price. Potentially, we could endow investors with a belief over  $\psi$ , which would then enable them to ascribe a probability to the outsider's presence. Such an assumption complicates the analysis without changing the qualitative nature of the insights.



This figure represents the equilibria we consider at time 2, for different values of  $\psi$  and  $\beta$ . The other parameters used to generate the figure are  $\theta_H = 0.9$ ,  $\theta_L = 0.55$ ,  $q = 0.4$ , and  $\alpha = 0.5$ .

Figure 2: Equilibria in the Continuation Game at Time 2 when Signals of Manager and Outsider Disagree

of the firm is then  $F_0 - c(\alpha)$ . Formally, the outsider acquires a stake  $\eta$  in the firm when the firm is valued at  $F_0 - c(\alpha)$ , and thus captures a fraction  $\eta$  of the improvement in cash flow that results from her intervention.

Suppose, instead, the outsider does acquire a stake in the firm and generates a signal about the project. The signal is then made public, and the game the game continues. Let  $F$  denote the expected cash flow from the project if the outsider intervenes, where the expectation is ex ante with respect to the outsider's signal; that is, the expectation is taken before the outsider knows her signal. The value of the firm if the outsider intervenes is then  $F - c(\alpha)$ .

The outsider incurs a fixed cost  $\tilde{\kappa}$  to acquire information about the project. Thus, she will enter if  $\eta(F - F_0) \geq \tilde{\kappa}$ , or  $F - F_0 \geq \frac{\tilde{\kappa}}{\eta}$ . Let  $\kappa = \frac{\tilde{\kappa}}{\eta}$  be the normalized cost to the outsider of generating a signal. Then, it is optimal for the outsider to enter if  $F - F_0 \geq \kappa$ .

Since the cost of the board's signal,  $c(\alpha)$ , is sunk at time 0, the outsider's decision depends only on the change in the expected cash flow from the project if she intervenes. The improvement in expected cash flow depends both on  $\psi$  and on the likelihood that the manager is overturned by the board when the signals of the manager and the outsider

disagree. Importantly, the expectation of cash flow in the next lemma is taken before the outsider has observed her own signal.

**Lemma 4.** *Suppose that the outsider intervenes, and, if  $s_M \neq s_E$ , a continuation equilibrium  $(\sigma, \gamma)$  is played at time 2, with  $\sigma_H = 1$ . Then, the expected cash flow from the project at time 1 before the outsider sees her signal is  $F = q[\theta_H - \gamma(\theta_H - \psi)] + (1 - q)[\psi - \sigma_L(1 - \alpha)(1 - \gamma)(\psi - \theta_L)]$ .*

Thus, the improvement in expected cash flow following the outsider's intervention is

$$F - F_0 = -q\gamma(\theta_H - \psi) + (1 - q)[1 - \sigma_L(1 - \alpha)(1 - \gamma)](\psi - \theta_L). \quad (7)$$

The external activist will intervene if and only if the improvement in expected cash flow exceeds  $\kappa$ ; i.e., if her signal is sufficiently precise. Her decision to intervene depends on the equilibrium being played at time 2. However, as seen in Section 3, the latter in turn depends on the level of agency conflict ( $\beta$ ) and on the precision of the outsider's signal ( $\psi$ ). Thus, the threshold value of  $\psi$  below which the activist will stay out depends on  $\beta$ .

We show that when  $\psi$  is low, the outsider stays out, regardless of the value of  $\beta$ . Similarly, for high values of  $\psi$ , the outsider always enters. However, there is also an intermediate region of  $\psi$ , in which the outsider only if  $\beta$  is sufficiently high (i.e., the agency conflict is sufficiently low). Define  $\psi_1 = \theta_L + \frac{\kappa}{1-q}$ , and  $\psi_2(\alpha) = \theta_L + \frac{\kappa}{\alpha(1-q)}$  if  $\alpha > 0$ . If  $\alpha = 0$ , let  $\psi_2(\alpha)$  be infinite. Finally, define a function  $\phi(\cdot)$  as follows:

$$\phi(\psi) = \frac{1}{1 + \frac{1-2\delta_L}{\theta_H - \theta_L} + \frac{(1-q)(\psi - \theta_L) - \kappa}{q\lambda_H(\theta_H - \theta_L)}}. \quad (8)$$

In the proof of Proposition 5, we show that  $\phi(\cdot)$  is a strictly decreasing, and hence invertible, function of  $\psi$ .

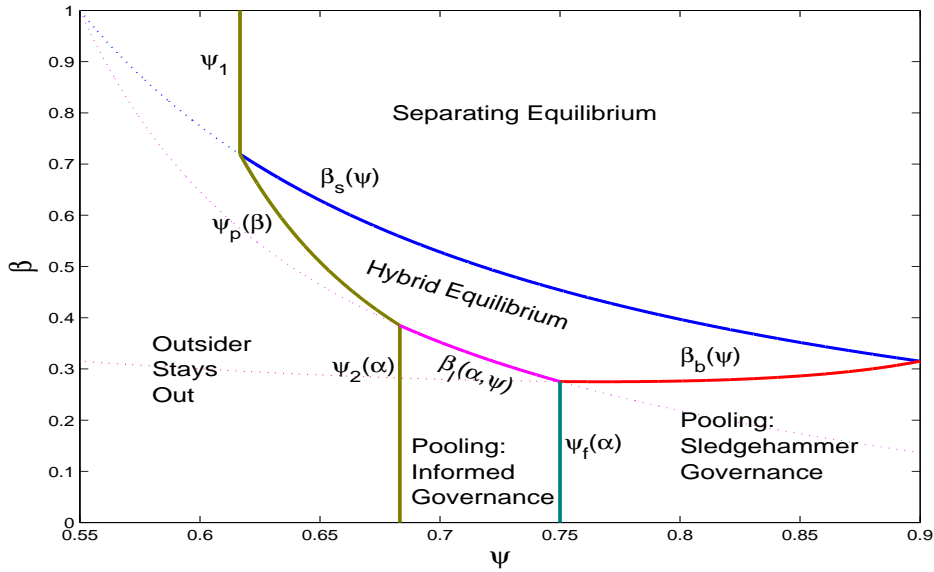
Provided the cost of entry,  $\kappa$ , is sufficiently low given other parameters, including  $\alpha$ , the outsider adopts the following strategy.

**Proposition 5.** *Suppose  $\kappa < \frac{\alpha q(1-q)(\theta_H - \theta_L)}{q + (1-\alpha)(1-q)}$ . Then, the outsider intervenes if  $\psi > \psi_2(\alpha)$  and stays out if  $\psi < \psi_1$ . If  $\psi \in [\psi_1, \psi_2(\alpha)]$ , the outsider intervenes if  $\beta > \phi(\psi)$  and stays out if  $\beta < \phi(\psi)$ .*

For particular values of  $\psi$  and  $\beta$ , the outsider is indifferent about intervening (e.g., when  $\psi = \psi_2$ , the outsider is indifferent if  $\beta \leq \phi(\psi_2)$ ). In the spirit of considering equilibria under which firm value is maximized, we assume the activist chooses to intervene in this case.

As  $\alpha$  increases,  $\psi_1$  and  $\phi^{-1}$  remain unchanged, whereas  $\psi_2$  shifts inward. An increase in  $\alpha$  implies that the board weeds out the low-type manager more often in a pooling equilibrium with informed governance, which increases the payoff to the outsider from generating her own information. Thus, the outsider is more likely to enter if she anticipates a pooling equilibrium with informed governance.

Figure 3 illustrates the optimal decision of the outsider for each value of  $\beta$  and  $\psi$ . The parameters used are the same as for Figure 2; that is,  $\theta_H = 0.9, \theta_L = 0.55, q = 0.4$ , and  $\alpha = 0.5$ . In addition, we set  $\kappa = 0.04$ .



This figure represents the optimal decision of the outsider at time 1 and the equilibria in the continuation game at time 2, for different values of the parameters  $\psi$  and  $\beta$ . The other parameters used to generate the figure are  $\theta_H = 0.9, \theta_L = 0.55, q = 0.4, \alpha = 0.5$ , and  $\kappa = 0.04$ .

Figure 3: Optimal Decision of Outsider for Different Values of  $\beta$  and  $\psi$

## 5 Optimal Level of Screening by the Board at Time 0

We now consider the board's optimal choice of screening intensity at time 0. As mentioned earlier, if the outsider stays out, the board allows the manager to proceed with his chosen project. Thus, screening has no value, and the board should set  $\alpha = 0$  in this case. If it anticipates the outsider will enter, the board chooses its screening intensity  $\alpha$  to maximize its overall payoff  $\Pi(\alpha) = F - c(\alpha)$ , where  $F$  is as defined in Lemma 4. The decision by the

board at this stage, of course, depends on the equilibrium to be played in the continuation game when the signals of the manager and the outsider disagree. If the signals agree the manager continues with his original choice of project, and the board remains passive.

We first show that if the continuation equilibrium at time 2 is a hybrid equilibrium, small changes in  $\alpha$  have no effect on the expected cash flow of the firm, so that the overall effect on profit depends only on changes in the cost of the screening technology. That is, a small change in the screening intensity of the board is completely unwound by a corresponding change in the strategy of the low-ability manager.

**Proposition 6.** *Suppose  $\psi \geq \psi_a(\alpha, \beta)$  and  $\beta \in (\max\{\beta_\ell(\alpha, \psi), \beta_b(\psi)\}, \beta_s(\psi))$ , so that the activist generates a signal and a hybrid equilibrium obtains in the continuation game at time 2. Then  $\Pi'(\alpha) = -c'(\alpha) < 0$ .*

Consider a value of  $\alpha$  that induces a hybrid equilibrium at time 1. All else equal, one would expect that an improvement in screening will improve the expected cash flow from the project, since the correct project is implemented more often. However, from Proposition 3 part (ii), we know that such an increase will be met by increased intransigence on the part of the low-ability manager. As we show in the proof of Proposition 6, these two effects exactly offset each other, so that the overall profit changes only to the extent that the cost of screening changes with  $\alpha$ .

Thus, if the board anticipates a hybrid equilibrium at time 1, it optimally chooses  $\alpha = 0$  at time 0. However, the equilibrium that obtains in the continuation game itself depends on the board's choice of  $\alpha$ . For a sufficiently high value of  $\alpha$  (high enough so that  $\beta_\ell(\alpha, \psi) = \beta$ ), the low type fights with probability 1 and a pooling equilibrium obtains. At this point, a further increase in  $\alpha$  cannot affect the strategy of the low-type manager. Therefore, the board may find it optimal to choose a high enough value of  $\alpha$  to induce a pooling equilibrium.

Suppose that the activist generates a signal and a pooling equilibrium with informed governance indeed obtains in the continuation game beginning at stage 3 of time 1. Consider the board's choice of  $\alpha$  at time 0. The optimal value of  $\alpha$  in this case must satisfy the following first-order condition:

$$c'(\alpha) = (1 - q)(\psi - \theta_L). \tag{9}$$

Let  $\alpha_c$  denote the level of  $\alpha$  that satisfies equation (9).

When it chooses the screening level  $\alpha_c$ , the board makes optimal use of the outsider's information. Since  $c(\cdot)$  is convex, it is immediate that  $\alpha_c$  increases as  $\psi$  increases. Higher values of  $\psi$  imply a greater benefit to overturning the low-type manager. If the outsider's

signal is sufficiently strong, the board may choose instead to completely delegate the decision to the activist by overturning the manager regardless of its signal. If it anticipates an equilibrium with sledgehammer governance at time 1, it should optimally choose  $\alpha = 0$  at time 0.

Define a threshold value  $\psi_g$  as the value of  $\psi$  that solves the implicit equation

$$\psi = \frac{q\theta_H + (1-q)(1-\alpha_c)\theta_L - c(\alpha_c)}{q + (1-q)(1-\alpha_c)}. \quad (10)$$

Then,  $\psi_g$  is the maximum value of  $\psi$  at which the board invests the cash flow maximizing amount  $\alpha_c$  in its screening technology.

**Lemma 5.** *Suppose the board anticipates that the outsider will enter and a pooling equilibrium will obtain at time 1. Then, if  $\psi \leq \psi_g$ , the board chooses  $\alpha = \alpha_c$  at time 0 and implements informed governance, with  $\gamma = 0$ . If  $\psi > \psi_g$ , the board chooses  $\alpha = 0$  at time 0 and implements sledgehammer governance with  $\gamma = 1$ .*

Next, we show that by choosing  $\alpha$  appropriately, the board can induce the activist to enter. The activist's signal increases firm value only if the board uses it to overturn the manager's decision. In a pooling equilibrium with informed governance, the likelihood that the board uses the activist's signal to improve decision-making increases with  $\alpha$ . Thus, there is a threshold value of  $\alpha$  above which the activist is willing to acquire information about the firm.

Define  $\alpha_e = \frac{\kappa}{(1-q)(\psi-\theta_L)}$ . It is immediate that  $\alpha_e$  declines in  $\psi$ , the precision of the outsider's signal.

**Lemma 6.** *Suppose the outsider anticipates a pooling equilibrium with informed governance (i.e.,  $\sigma_L = 1$  and  $\gamma = 0$ ). Then, she enters if and only if  $\alpha \geq \alpha_e$ .*

If  $\psi$  is very low, the board does not find it worthwhile to choose  $\alpha = \alpha_e$  and induce the outsider to generate a signal. Let  $\psi_3 = \theta_L + \frac{\kappa}{(1-q)c^{-1}(\kappa)}$  and define  $\beta_3 = \phi(\psi_3)$ . Recall that  $\psi_1 = \frac{\kappa}{1-q}$ . As we show shortly,  $\psi_1$  and  $\psi_3$  help to define a lower boundary for  $\psi$ , below which, in the equilibrium of the overall game, the board does not invest in learning about the manager's type and the outsider stays out.

Next, we define a threshold value of  $\beta$ , at which the board is indifferent between choosing the cash flow maximizing investment  $\alpha_c$  and inducing a pooling equilibrium with informed

governance, and choosing  $\alpha = 0$  and inducing a hybrid equilibrium at time 2. Define

$$\beta_c(\psi) = \frac{1}{1 + \frac{1-2\theta_L}{\theta_H-\theta_L} \left[ 1 + \frac{(1-q)\lambda_L}{q\lambda_H} \left\{ 1 - \alpha_c + \frac{c(\alpha_c)}{c'(\alpha_c)} \right\} \right]} \quad (11)$$

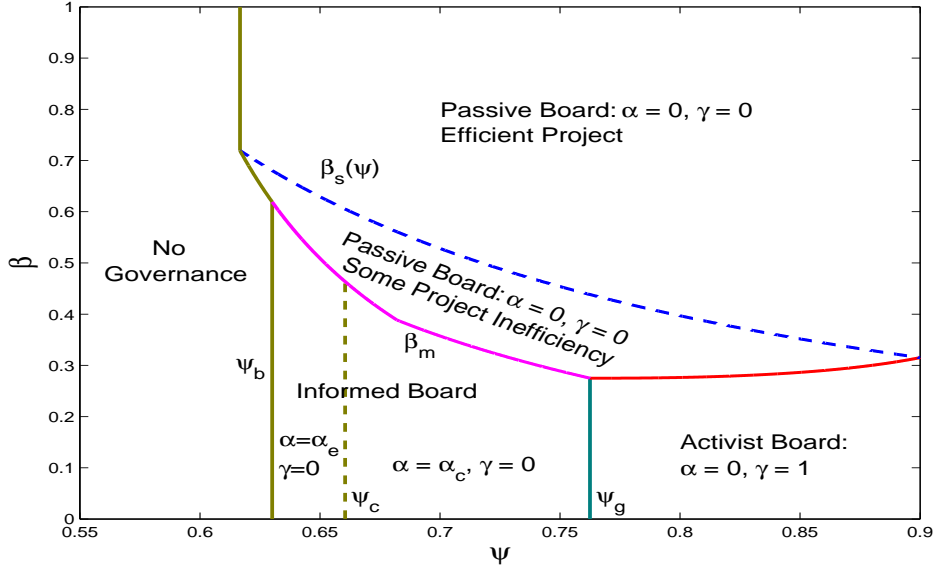
Define  $\beta_m(\psi) = \max\{\phi(\psi), \beta_c(\psi), \beta_b(\psi)\}$ . As we show in the proof of the next proposition,  $\beta_m$  equals  $\phi(\psi)$  for low values of  $\psi$  and  $\beta_b$  for high values of  $\psi$ . If  $\kappa$  is not too high, there also exists an intermediate range of  $\psi$  for which  $\beta_m$  equals  $\beta_c(\psi)$ . Finally, let  $\kappa_1$  be the strictly positive solution to  $\kappa = c\left(\frac{\kappa}{q(1-q)(\theta_H-\theta_L)}\right)$ , and let  $\kappa_2 = \psi_g - q\theta_H - (1-q)\theta_L$ .

**Proposition 7.** *Suppose  $\kappa < \min\{\kappa_1, \kappa_2\}$ . Then,*

- (i) *If  $\psi < \psi_1$ , or if  $\psi \in [\psi_1, \psi_3)$  and  $\beta < \phi(\psi)$ , the equilibrium is characterized by no governance. The board chooses  $\alpha = 0$ , the outsider stays out, and the board allows the manager to proceed at time 2; that is,  $\alpha = \xi = \gamma = 0$ .*
- (ii) *If  $\psi \geq \psi_1$  and  $\beta \geq \beta_m(\psi)$ , the board continues to be completely passive, with  $\alpha = \gamma = 0$ . However, the outsider enters, so that  $\xi = 1$ . Either a separating or a hybrid equilibrium is played at time 2.*
- (iii) *If  $\psi \geq \psi_3$  and  $\beta < \beta_m$ , there exists a  $\psi_c \in (\psi_3, \psi_g)$  such that:*
  - (a) *If  $\psi \leq \psi_g$ , the board is informed, choosing  $\alpha = \alpha_e$  when  $\psi \in [\psi_3, \psi_c)$  and  $\alpha = \alpha_c$  when  $\psi \in (\psi_c, \psi_g]$ . In both cases, the outsider enters, so that  $\xi = 1$ , and a pooling equilibrium with informed governance is played, with  $\gamma = 0$ .*
  - (b) *If  $\psi \in (\psi_g, \theta_H)$ , the board chooses  $\alpha = 0$ . The outsider enters, so that  $\xi = 1$ , and a pooling equilibrium with sledgehammer governance is played, with  $\gamma = 1$ .*

Figure 4 illustrates the equilibrium of the overall game for different values of  $\beta$  and  $\psi$ . The parameters used are the same as for Figures 2 and 3; that is,  $\theta_H = 0.9$ ,  $\theta_L = 0.55$ ,  $q = 0.4$ , and  $\kappa = 0.04$ . Now, however, instead of fixing the value of  $\alpha$ , we allow  $\alpha$  to be chosen optimally by the board. Thus  $\alpha$  will vary with the values of  $\beta$  and  $\psi$ . We assume a cost function for the board's signal of  $c(\alpha) = 0.1\alpha^5$ . While this cost function does not satisfy the condition  $\lim_{\alpha \rightarrow 1} c'(\alpha) = \infty$ , in the example the optimal level of  $\alpha$  remains strictly below one.

When  $\psi < \psi_b(\beta)$ , the outsider stays out, so the board cannot gain from generating a signal about the manager. Hence, there is no governance in this region. When  $\beta \geq \beta_m$  and  $\psi \geq \psi_b(\beta)$ , the outsider enters, but the board is optimally passive. It chooses to set  $\alpha = 0$ , and allows the manager to choose the project. If  $\beta \geq \beta_s$ , this achieves the first-best



This figure represents the equilibria that occur in the overall game for different values of the parameters  $\psi$  and  $\beta$ . The other parameters used to generate the figure are  $\theta_H = 0.9, \theta_L = 0.55, q = 0.4, c(\alpha) = 0.1\alpha^5$ , and  $\kappa = 0.04$ .

Figure 4: Equilibria in the Overall Game for Different Values of  $\beta$  and  $\psi$ , when the Outsider Acts Optimally

outcome, since the manager optimally chooses the value-maximizing project. However, if  $\beta \in (\beta_m, \beta_s)$ , the low-type manager fights with positive probability, resulting in some inefficiency in project choice. Nevertheless, as we have shown, it is optimal for the board to be passive.

It is optimal for the board to invest in its screening technology if  $\beta < \beta_m$  and  $\psi \in (\psi_b(\beta), \psi_g)$ . If  $\psi > \psi_c$ , it chooses  $\alpha = \alpha_c$ , which is optimal purely from a cash flow viewpoint. If  $\psi < \psi_c$ , the board has to over-invest in screening to induce the outsider to enter, and chooses  $\alpha = \alpha_e$ .

The board's optimal policy exhibits several discontinuities when  $\psi \geq \psi_b(\beta)$ . First, suppose  $\psi \in (\psi_b(\beta), \psi_c)$  and  $\beta < \beta_m$ . Consider an increase in  $\beta$  to  $\beta_m$ . At this point, the board switches from informed governance, with  $\alpha \geq \alpha_c$ , to being completely passive. Second, suppose  $\psi > \psi_g$ , and consider a similar increase in  $\beta$  to  $\beta_m$ . The board now switches from extreme activism in the form of sledgehammer governance, in which the manager is always overturned, to complete passivity. Finally, consider the effect of an increase in  $\psi$  when  $\beta < \beta_m$ . When  $\psi$  increases to  $\psi_g$ , the board's investment in screening drops from  $\alpha_c$

to zero. Screening is substituted out in favor of extreme activism.

## 5.1 Internal and External Governance: Substitutes or Complements?

The relationship between internal governance and external governance is complex in our model. The outsider's signal represents information generated outside the firm that nevertheless has an impact on the manager's decisions. Thus, the precision of the outsider's signal,  $\psi$ , is a measure of the strength of external governance. Internal governance is carried out by the board, and is represented by both the screening intensity  $\alpha$  and the overturning probability  $\gamma$ . Finally,  $\beta$  captures the extent of the agency conflict at the firm. If  $\beta$  is low, the manager is overly concerned about his reputation rather than the value of the firm, and the agency conflict is high.

Consider the case in which the agency conflict is high; that is,  $\beta$  is low. If the outsider's signal is imprecise, she will stay out, so that the board is passive as well. As the precision of the outsider's signal improves, she switches over to generating a signal, thus providing external governance. At this threshold, the board sets  $\alpha = \alpha_e$ , which declines in  $\psi$ . Since  $\alpha_e > \alpha_c$  for each value of  $\psi$ , the board over-invests in screening relative to the level that ensures optimal use of the outsider's information. Such an over-investment induces the outsider to generate information about the firm.<sup>13</sup> Further, the board implements informed governance, overturning the manager only when it obtains the low signal. The overall probability that the manager is overturned is monotonic in  $\alpha$ , and hence also declining in  $\psi$  over this region. Hence, for  $\psi \in [\psi_b(\beta), \psi_c]$ , internal and external governance are substitutes.

However, if  $\psi$  lies between  $\psi_c$  and  $\psi_g$ , the board sets  $\alpha = \alpha_c$ , which is *increasing* in  $\psi$ . The intuition here is that the value to the board of overturning the low type increases as the precision of the outsider's signal increases. Thus, it invests a greater amount in its screening technology. The board continues to implement informed governance, so the overturning probability is also increasing in  $\psi$ . Thus, internal and external governance are complements in this region.

Finally, if  $\psi > \psi_g$  and  $\beta$  is low, the board does not screen the manager, and simply acts on the public signal in deciding whether or not to overrule managerial decisions. In this sense, external governance completely substitutes for internal governance over this region of the parameter space.

**Proposition 8.** *Suppose  $\psi \geq \psi_b(\beta)$  and  $\beta < \beta_m(\psi)$ . Then, the screening intensity of the board,  $\alpha$ , decreases with the precision of the external signal,  $\psi$ , when  $\psi \leq \psi_c$ . However, for*

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<sup>13</sup>In the spirit of Aghion and Tirole (1997), choosing a high  $\alpha$  amounts to a commitment to effectively cede control to the outsider when the board obtains a negative signal about the manager.

$\psi \in (\psi_c, \psi_g]$ , the screening intensity of the board increases with the precision of the external signal.

Thus, while large changes in the strength of external governance result in external governance substituting for internal governance, small changes in the strength of external governance can have complementary effects on internal governance. Overall, therefore, we find a non-monotone relationship between external and internal governance. Depending on the strength of external governance, it may either complement or substitute for internal governance. Our results therefore suggest that corporate governance indices, such as those of Gompers, Ishii and Metrick (2003) and Bebchuk, Cohen, and Ferrell (2004) must be interpreted with caution. A higher index value may not imply better governance, but may instead just reflect the severity of the agency problem at the firm.

## 6 Conclusion

We examine optimal internal corporate governance when a manager is concerned about his reputation and faces potential discipline from the market for corporate control, where the latter is represented by an activist shareholder. Reputational concerns may cause a manager to deviate from the value-maximizing action; their scale indicates the degree of the conflict between the manager and shareholders. As we show, the optimal internal governance strategy implemented by the board depends both on the potential for agency conflict and the strength of external governance.

It is immediate that when the agency conflict is minimal, the board does not need to act. However, the board also ignores a moderate agency conflict, even though the manager sometimes chooses a project that is sub-optimal for shareholders. In this situation, by increasing its effort on screening, the board can identify an incorrect project more often. However, an increased amount of screening exacerbates a low-type manager's reputational concerns, and as a result leads to him choosing the sub-optimal project more often. In equilibrium, this leads to the board optimally choosing to be completely passive even when the manager is moderately conscious of his reputation.

As the manager becomes more conscious of his reputation, at some point the optimal level of governance shifts discontinuously. Informed governance by the board replaces passivity at this point. The board invests a finite amount in screening, and overturns the manager if it determines he is in the incorrect project. Finally, we show that the relationship between external and internal governance is non-monotone. When external governance is weak, the board needs to over-invest in internal governance, to induce the outsider to play a role. Beyond a point, external governance then becomes a complement to internal gov-

ernance. When external governance is strong, the board relies completely on the outsider and adopts an interventionist policy.

In our model, the board plays a crucial role in deciding whether control over the firm's strategy should rest with management or the outsider. Overall, therefore, our work points to a role for the board as an arbitrator in disputes between the managers and activist shareholders.

## 7 Appendix

### Proof of Lemma 1

(i) Suppose  $s_E = B$  and the manager fights. Then, the manager must have obtained signal  $s_M = A$ .

Now, suppose the board obtains signal  $H$ . At time 1, the cost  $c(\alpha)$  is sunk, and can be ignored. Ignoring  $c(\alpha)$ , if the board allows the manager to continue with project  $A$ , it obtains an expected payoff  $\mu_f(\alpha)\delta_H + (1 - \mu_f(\alpha))\delta_L = \delta_L + \mu_f(\alpha)(\delta_H - \delta_L)$ . If it overturns the manager and implements project  $B$ , the board's expected payoff is  $\mu_f(\alpha)(1 - \delta_H) + (1 - \mu_f(\alpha))(1 - \delta_L) = 1 - \delta_L - \mu_f(\alpha)(\delta_H - \delta_L)$ . Therefore, it is a best response for the board to overturn the manager if and only if

$$1 - \delta_L - \mu_f(\alpha)(\delta_H - \delta_L) \geq \delta_L + \mu_f(\alpha)(\delta_H - \delta_L), \quad (12)$$

or  $\mu_f(\alpha) \leq \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)}$ . The statement of part (i) of the Lemma follows immediately.

(ii) Consider the high-type manager. If he concedes, by assumption the board allows the concession to stand, so project  $B$  is implemented. Further, investors believe he has an expected type  $\mu_c(\alpha)\theta_H + (1 - \theta)\theta_L = \theta_L + \mu_c(\alpha)(\theta_H - \theta_L)$ . Thus, his expected payoff is  $\beta(1 - \delta_H) + (1 - \beta)[\theta_L + \mu_c(\alpha)(\theta_H - \theta_L)]$ .

If he fights, the board obtains signal  $H$ . Thus, with probability  $(1 - \gamma)$ , project  $A$  is undertaken, and with probability  $\gamma$  he is overturned and project  $B$  is undertaken. The expected cash flow from the project (ignoring the sunk cost  $c(\alpha)$ ) is  $(1 - \gamma)\delta_H + \gamma(1 - \delta_H)$ .

Now, consider the reputational component of his payoff. If he fights, investors believe he has type  $H$  with probability  $\mu_f(\alpha)$ . Thus, their posterior expectation over his type is  $\theta_L + \mu_f(\alpha)(\theta_H - \theta_L)$ . Therefore, if he fights, the overall expected payoff of the type  $H$  manager is  $\beta[(1 - \gamma)\delta_H + \gamma(1 - \delta_H)] + (1 - \beta)[\theta_L + \mu_f(\alpha)(\theta_H - \theta_L)]$ . It is a best response to fight if and only if

$$\begin{aligned} \beta[(1 - \gamma)\delta_H + \gamma(1 - \delta_H)] + (1 - \beta)[\theta_L + \mu_f(\alpha)(\theta_H - \theta_L)] &\geq \\ \beta(1 - \delta_H) + (1 - \beta)[\theta_L + \mu_c(\alpha)(\theta_H - \theta_L)], &\quad (13) \end{aligned}$$

or  $(1 - \beta)[\mu_f(\alpha) - \mu_c(\alpha)](\theta_H - \theta_L) \geq \beta(1 - \gamma)(1 - 2\delta_H)$ . The last inequality reduces to

$$\mu_f(\alpha) \geq \mu_c(\alpha) + (1 - \gamma)\frac{\beta}{1 - \beta}\frac{1 - 2\delta_H}{\theta_H - \theta_L}. \quad (14)$$

Next, consider the low-type manager. If he concedes, project  $B$  is implemented. With probability  $\alpha$ , the board will obtain signal  $L$ , and investors will recognize his type as  $\theta_L$ . With probability  $(1 - \alpha)$ , investors will have a posterior expectation over his type of  $\theta_L + \mu_c(\alpha)(\theta_H - \theta_L)$ . Thus, his expected payoff if he concedes is  $\beta(1 - \delta_L) + (1 - \beta)[\alpha\theta_L + (1 -$

$$\alpha\{\theta_L + \mu_c(\alpha)(\theta_H - \theta_L)\} = \beta(1 - \delta_L) + (1 - \beta)[\theta_L + (1 - \alpha)\mu_c(\alpha)(\theta_H - \theta_L)].$$

If he fights, with probability  $\alpha$  he is revealed to have type  $\theta_L$  and project  $B$  is implemented. With probability  $(1 - \alpha)$ , investors' posterior expectation over his type is  $\theta_L + \mu_c(\alpha)(\theta_H - \theta_L)$ , and project  $A$  is continued with probability  $\gamma$ . Thus, the expected cash flow component of his payoff is  $\alpha(1 - \delta_L) + (1 - \alpha)[\gamma(1 - \delta_L) + \gamma(1 - \delta_L)]$ . The reputational component is  $\alpha\theta_L + (1 - \alpha)[\theta_L + \mu_f(\alpha)(\theta_H - \theta_L)] = \theta_L + (1 - \alpha)\mu_f(\alpha)(\theta_H - \theta_L)$ . Therefore, it is a best response to fight if and only if

$$\begin{aligned} \beta[\alpha(1 - \delta_L) + (1 - \alpha)[\gamma(1 - \delta_L) + (1 - \gamma)\delta_L] + (1 - \beta)[\theta_L + (1 - \alpha)\mu_f(\alpha)(\theta_H - \theta_L)] \\ \geq \beta(1 - \delta_L) + (1 - \beta)[\theta_L + (1 - \alpha)\mu_c(\alpha)(\theta_H - \theta_L)], \end{aligned} \quad (15)$$

or  $(1 - \alpha)(1 - \beta)[\mu_f(\alpha) - \mu_c(\alpha)](\theta_H - \theta_L) \geq (1 - \alpha)\beta(1 - \gamma)(1 - 2\delta_L)$ . The last inequality reduces to

$$\mu_f(\alpha) \geq \mu_c(\alpha) + (1 - \gamma)\frac{\beta}{1 - \beta}\frac{1 - 2\delta_L}{\theta_H - \theta_L}. \quad (16)$$

The statement of part (ii) of the Lemma follows from the inequalities (14) and (16). ■

## Proof of Lemma 2

(i) Suppose  $\gamma < 1$ . Further, suppose that  $\sigma_L > 0$ . Then, from Lemma 1, part (ii), it follows that

$$\mu_f(\alpha) \geq \mu_c(\alpha) + (1 - \gamma)\frac{\beta}{1 - \beta}\frac{1 - 2\delta_L}{\theta_H - \theta_L}.$$

Since  $\delta_H > \delta_L$ , it follows that

$$\mu_f(\alpha) > \mu_c(\alpha) + (1 - \gamma)\frac{\beta}{1 - \beta}\frac{1 - 2\delta_H}{\theta_H - \theta_L}, \quad (17)$$

so that the high-type manager strictly prefers to fight; i.e.,  $\sigma_H = 1$ .

Next, suppose  $\sigma_L = 0$ , and  $\sigma_H \in (0, 1)$ . Since only the high-type manager fights, Bayes' rule implies that  $\mu_f(\alpha) = 1$ . Further, note that  $\delta_H > \frac{1}{2}$ , so  $1 - 2\delta_H < 0$ . It follows that, for any value of  $\mu_c(\alpha) \leq 1$ , condition (17) is again satisfied. Then, the high-type manager must fight with probability one, contradicting the conjecture that  $\sigma_H \in (0, 1)$ . Therefore, if  $\sigma_L = 0$ , it must be that either  $\sigma_H = 0$  or  $\sigma_H = 1$ .

(ii) Suppose  $\gamma = 0$ . Then, both types of manager strictly prefer to fight if  $\mu_f(\alpha) > \mu_c(\alpha)$ , and to concede if  $\mu_f(\alpha) < \mu_c(\alpha)$ . Suppose  $\sigma_H \neq \sigma_L$  in equilibrium. Then, both the "concede" and "fight" information sets are reached along the path of play, so Bayes' rule pins down the beliefs  $\mu_f(\alpha)$  and  $\mu_c(\alpha)$ . From equations (1) and (2), it is straightforward to see that when  $\sigma_H \neq \sigma_L$ , it cannot be that  $\mu_f(\alpha) = \mu_c(\alpha)$ . Therefore, either both types

strictly prefer to fight, or both types strictly prefer to concede. In either case,  $\sigma_H = \sigma_L$ , contradicting the conjecture that  $\sigma_H \neq \sigma_L$ . ■

### Proof of Proposition 1

Since only the high-type manager fights,  $\mu_f(\alpha) = 1$  and  $\mu_c(\alpha) = 0$ . Thus, it follows that  $\mu_f(\alpha) > \mu_c(\alpha) + (1 - \gamma) \frac{\beta}{1 - \beta} \frac{1 - 2\delta_H}{\theta_H - \theta_L}$ , so that from Lemma 1 part (ii), it is a best response for the high-type manager to fight; i.e., set  $\sigma_H = 1$ . Further, note that  $\delta_H > \frac{1}{2}$  implies that  $\frac{1 - 2\delta_L}{2(\delta_H - \delta_L)} < 1$ , so that  $\mu_f(\alpha) > \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)}$ . Therefore, from Lemma 1 part (i) it is a best response for the board to choose  $\gamma = 0$ .

Finally, consider the low-type manager. It is a best response for him to set  $\sigma_L = 0$  (i.e., to concede with probability one) if and only if  $\mu_f(\alpha) \leq \mu_c(\alpha) + (1 - \gamma) \frac{\beta}{1 - \beta} \frac{1 - 2\delta_L}{\theta_H - \theta_L}$ . Substituting  $\mu_f(\alpha) = 1$ ,  $\mu_c(\alpha) = 0$  and  $\gamma = 0$ , this inequality reduces to

$$\frac{\beta}{1 - \beta} \frac{1 - 2\delta_L}{\theta_H - \theta_L} \geq 1, \quad (18)$$

or  $\beta \geq \beta_s(\psi)$ . ■

### Proof of Proposition 2

Given the equilibrium strategies of the managers,  $\mu_f(\alpha)$  is pinned down by Bayes' rule, and may be written as  $\mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L}$ . Since neither type of manager concedes, the "concede" information set is reached with probability zero. Assign the belief  $\mu_c(\alpha) = 0$  at this information set. That is, if the manager concedes, investors believe he has the low type with probability 1. Finally, note that  $\gamma = 0$  in the conjectured equilibrium.

Then, from Lemma 1 (ii), it is a best response for the low-type manager to fight if and only if  $\mu_f(\alpha) \geq \frac{\beta}{1 - \beta} \frac{1 - 2\delta_L}{\theta_H - \theta_L}$ . In other words, it is a best response to set  $\sigma_L = 1$  if and only if

$$\frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L} \geq \frac{\beta}{1 - \beta} \frac{1 - 2\delta_L}{\theta_H - \theta_L}, \quad (19)$$

or  $\beta \leq \beta_\ell(\alpha, \psi)$ . Thus, the low-type manager sets  $\sigma_L = 1$ .

Next, consider the high-type manager. Since  $\mu_c(\alpha) = 0 < \mu_f(\alpha)$ , it is a best response for him to set  $\sigma_H = 1$ .

Finally, consider the best response of the board. From Lemma 1 (i), The board should set  $\gamma = 0$  if and only if  $\mu_f(\alpha) \geq \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)}$ . Since  $\sigma_H = \sigma_L = 1$ ,  $\mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L}$ . Therefore, the board should set  $\gamma = 0$  if and only if

$$\frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L} \geq \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)}. \quad (20)$$

Cross-multiplying and rearranging terms, this inequality is equivalent to

$$q\lambda_H(2\delta_H - 1) \geq (1 - q)(1 - \alpha)\lambda_L(1 - 2\delta_L). \quad (21)$$

Now, note that for each  $i = H, L$ ,  $\lambda_i = \theta_i(1 - \psi) + \psi(1 - \theta_i)$  and  $\delta_i = \frac{\theta_i(1 - \psi)}{\lambda_i}$ . Hence, it follows that  $\lambda_H(2\delta_H - 1) = \theta_H - \psi$ , and  $\lambda_L(1 - 2\delta_L) = \psi - \theta_L$ . Making these substitutions, the inequality (21) may be re-written as

$$q(\theta_H - \psi) \geq (1 - q)(1 - \alpha)(\psi - \theta_L), \quad (22)$$

$$\text{Or, } \psi \leq \frac{q\theta_H + (1 - q)(1 - \alpha)\theta_L}{q + (1 - q)(1 - \alpha)} = \psi_f(\alpha). \quad (23)$$

Hence, it is optimal for the board to set  $\gamma = 0$  if and only if  $\psi \leq \psi_f(\alpha)$ .  $\blacksquare$

### Proof of Proposition 3

(i) First, consider the high-type manager. It is a best response for him to fight if  $\mu_f(\alpha) \geq \mu_c(\alpha) + (1 - \gamma)\frac{\beta}{1 - \beta}\frac{1 - 2\delta_H}{\theta_H - \theta_L}$ . Since  $\sigma_H = 1$  and  $\sigma_L \in (0, 1)$ , it follows that  $\mu_c(\alpha) = 0$  and  $\mu_f(\alpha) > 0$ . Since  $\delta_H > \frac{1}{2}$ , the inequality is satisfied. Hence, it is a best response for type  $\theta_H$  to set  $\sigma_H = 1$ .

Next, consider the low-type manager. Since  $\mu_c(\alpha) = 0$  and  $\gamma = 0$ , it is a best response for him to mix between fighting and conceding if and only if

$$\mu_f(\alpha) = \frac{\beta}{1 - \beta}\frac{1 - 2\delta_L}{\theta_H - \theta_L}. \quad (24)$$

Since  $\sigma_H = 1$ , we can further write

$$\mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H + (1 - \alpha)(1 - q)\lambda_L\sigma_L}. \quad (25)$$

Equating the right-hand sides of (24) and (25), we obtain

$$\sigma_L = \frac{q\lambda_H}{(1 - \alpha)(1 - q)\lambda_L} \left[ \frac{1 - \beta}{\beta} \frac{\theta_H - \theta_L}{1 - 2\delta_L} - 1 \right]. \quad (26)$$

Therefore,  $\sigma_L > 0$  requires  $\beta < \beta_s(\psi)$ , and  $\sigma_L < 1$  requires  $\beta > \frac{1}{1 + \frac{1 - 2\delta_L}{\theta_H - \theta_L} \left[ 1 + \frac{(1 - \alpha)(1 - q)\lambda_L}{q\lambda_H} \right]} = \beta_\ell(\alpha, \psi)$ . That is, if  $\beta \in (\beta_\ell(\alpha, \psi), \beta_s(\psi))$ ,  $\sigma_L$  as defined is strictly between 0 and 1 and constitutes a best response for type  $\theta_L$ .

Finally, consider the action of the board when it obtains signal  $H$ . It is a best response for the board to set  $\gamma = 0$  if

$$\mu_f(\alpha) \geq \frac{1 - 2\delta_L}{2(\delta_H - \delta_L)}, \quad (27)$$

Substituting the expression in (24) for  $\mu_f(\alpha)$ , the above inequality holds if and only if  $\beta \geq \frac{1}{1+2\frac{\delta_H-\delta_L}{\theta_H-\theta_L}} = \beta_b$ . Hence, if  $\beta > \beta_b(\psi)$ , the board maximizes its payoff by setting  $\gamma = 0$ .

Therefore, if  $\beta > \max\{\beta_\ell(\alpha, \psi), \beta_b(\psi)\}$  and  $\beta < \beta_s(\psi)$ , a hybrid equilibrium exists in the continuation game, with  $\sigma_H = 1$ ,  $\sigma_L \in (0, 1)$ , and  $\gamma = 0$ .

(ii) Consider the expression for  $\sigma_L$  in equation (26). It is immediate that as  $\alpha$  increases,  $\sigma_L$  increases as well. ■

#### Proof of Proposition 4

Suppose there is an equilibrium in which  $\gamma = 1$ . From Lemma 1, it must be that  $\sigma_H = \sigma_L$ . Now consider any value of  $\sigma_L \in (0, 1)$ , and let  $\sigma_H = \sigma_L$ . From Bayes' rule,  $\mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H+(1-\alpha)(1-q)\lambda_L}$ . Unless  $\sigma_H = \sigma_L = 1$ , it follows again from Bayes' rule that  $\mu_c(\alpha) = \mu_f(\alpha)$ . If  $\sigma_H = \sigma_L = 1$ , assign  $\mu_c(\alpha) = \mu_f(\alpha)$ .

Now, from Lemma 1, when  $\gamma = 1$  and  $\mu_f(\alpha) = \mu_c(\alpha)$ , each type of manager is indifferent between fighting and conceding. Thus, each type of manager is playing a best response. It only remains to be shown that the board plays a best response given the strategies of each type of manager.

From Lemma 1, it is optimal for the board to choose  $\gamma = 1$  if and only if  $\mu_f(\alpha) \leq \frac{1-2\delta_L}{2(\delta_H-\delta_L)}$ . Note that  $\mu_f(\alpha) = \frac{q\lambda_H}{q\lambda_H+(1-\alpha)(1-q)\lambda_L}$ . Then, from the proof of Proposition 2, it follows that the board should set  $\gamma = 1$  if and only if  $\psi \geq \psi_f(\alpha)$ . ■

#### Proof of Lemma 3

Let  $z$  denote the posterior probability the manager has type  $\theta_H$ , conditional on  $s_M \neq s_E$ . Then,  $z = \frac{q\lambda_H}{q\lambda_H+(1-q)\lambda_L}$ . Suppose the continuation equilibrium at time 2 is  $(\sigma, \gamma)$ . Then, the expected payoff of the board is the expected cash flow from the project less the cost of the screening procedure,  $c(\alpha)$ . That is,

$$P = z[\gamma(1 - \delta_H) + (1 - \gamma)\delta_H] + (1 - z)[(1 - \delta_L) - \sigma_L(1 - \alpha)(1 - \gamma)(1 - 2\delta_L)] - c(\alpha).$$

Suppose  $\psi \leq \psi_f(\alpha)$  and  $\beta > \beta_s$ . Then, from Propositions 1 and 4, both a separating equilibrium and a pooling equilibrium with sledgehammer governance exist. In the separating equilibrium,  $\sigma_L = 0$  and  $\gamma = 0$ . Thus, the board's payoff is

$$\bar{P} = z\delta_H + (1 - z)(1 - \delta_L) - c(\alpha). \quad (28)$$

In the sledgehammer equilibrium,  $\sigma_L = 1$  and  $\gamma = 1$ . Thus, the board's payoff is

$$\tilde{P} = z(1 - \delta_H) + (1 - z)(1 - \delta_L) - c(\alpha). \quad (29)$$

Now,  $\bar{P} - \tilde{P} = z(2\delta_H - 1) > 0$ , since  $\delta_H > 1/2$ .

Now suppose that  $\psi > \psi_f(\alpha)$  and  $\beta > \beta_b$ . Then, from Propositions 3 and 4, both the hybrid and sledgehammer equilibria exist. In this equilibrium,  $\gamma = 1$ , and the exact expression for  $\sigma_L$  is shown in equation (26). From equation (26),  $\sigma_L(1 - \alpha) = \frac{q\lambda_H}{(-q)\lambda_L} \left[ \frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{1-2\delta_L} - 1 \right]$ .

Therefore, the board's payoff from the hybrid equilibrium is

$$\hat{P} = z\delta_H + (1-z)(1-\delta_L) + z\left[\frac{1-\beta}{\beta} \theta_H - \theta_L - (1-2\delta_L)\right] - c(\alpha). \quad (30)$$

Subtracting the board's expected payoff in the equilibrium with sledgehammer governance, we have  $\hat{P} - \tilde{P} = z[2(\delta_H - \delta_L) - \frac{1-\beta}{\beta}(\theta_H - \theta_L)]$ . It follows that the condition  $\hat{P} \geq \tilde{P}$  is equivalent to the condition  $\beta \geq \beta_b(\psi)$ . ■

#### Proof of Lemma 4

We first prove the following claim.

*Claim:* Suppose the manager receives signal  $s_M \in \{A, B\}$ , the activist enters, and, in the continuation equilibrium, the project favored by the manager's signal is undertaken with probability  $p$  whenever  $s_E \neq s_M$ . Then, the expected cash flow from the project before the activist observes her signal is  $p\theta_i + (1-p)\psi$ .

*Proof of Claim:*

There are two cases to consider. First, the signal of the activist investor agrees with the manager's signal with probability  $1 - \lambda_i = \theta_i\psi + (1 - \theta_i)(1 - \psi)$ . If  $s_M = s_E = Y \in \{A, B\}$ , the true state is  $x_Y$  with conditional probability  $\frac{\theta_i\psi}{1-\lambda_i}$ . Hence, the expected cash flow in this case is  $\frac{\theta_i\psi}{1-\lambda_i}$ .

Next, suppose  $s_E \neq s_M$ . This event occurs with probability  $\lambda_i = \theta_i(1 - \psi) + \psi_p(1 - \theta_i)$ . If the project favored by the manager's signal is undertaken, the expected cash flow is  $\delta_i = \frac{\theta_i(1-\psi)}{\lambda_i}$ . If the project favored by the outsider's signal is undertaken, the expected cash flow is  $1 - \delta_i = \frac{\psi(1-\theta_i)}{\lambda_i}$ .

Now, when  $s_E \neq s_M$ , the project favored by the manager's signal is undertaken with probability  $p$ . Thus, before the activist observes her signal, the expected cash flow from the project is

$$(1 - \lambda_i) \frac{\theta_i\psi}{1 - \lambda_i} + \lambda_i \frac{p\theta_i(1 - \psi) + (1 - p)\psi(1 - \theta_i)}{\lambda_i} = p\theta_i + (1 - p)\psi.$$

This proves the claim.

Now, we return to the proof of the Lemma. Suppose the manager has type  $\theta_H$  and  $s_M \neq s_E$ . The high-type manager fights with probability 1, and the project favored by the

manager's signal is undertaken with probability  $1 - \gamma$ . Hence, the expected cash flow from the project is  $(1 - \gamma)\theta_H + \gamma(\psi) = \theta_H - \gamma(\theta_H - \psi)$ .

Next, suppose the manager has type  $\theta_L$  and  $s_M \neq s_E$ . With probability  $1 - \sigma_L$ , the manager concedes, and the project favored by the public signal is undertaken. With probability  $\sigma_L$ , he fights, and is overturned with probability  $\alpha + (1 - \alpha)\gamma = \gamma + \alpha(1 - \gamma)$ . Hence, with cumulative probability  $1 - \sigma_L + \sigma_L\{\gamma + \alpha(1 - \gamma)\}$ , the project favored by the public signal is undertaken. Note that this probability may be written as  $1 - \sigma_L\{1 - \gamma - \alpha(1 - \gamma)\}$ . With probability  $\sigma_L\{1 - \gamma - \alpha(1 - \gamma)\}$  the project favored by the manager's signal is undertaken. Thus, the expected cash flow from the project is  $\psi - \sigma_L\{1 - \gamma - \alpha(1 - \gamma)\}(\psi - \theta_L)$ .

Hence, the overall expected cash flow from the project is

$$F = q[\theta_H - \gamma(\theta_H - \psi)] + (1 - q)[\psi - \sigma_L\{1 - \gamma - \alpha(1 - \gamma)\}(\psi - \theta_L)]. \quad (31)$$

■

### Proof of Proposition 5

From equation (7), using the appropriate values of  $\gamma$  in each case, it follows that the payoff improvement if the outsider intervenes is  $\Delta_S(\psi) = (1 - q)(\psi - \theta_L)$  if the continuation equilibrium is separating,  $\Delta_Y(\psi, \alpha, \sigma_L) = (1 - q)[1 - \sigma_L(1 - \alpha)](\psi - \theta_L)$  if it is hybrid,  $\Delta_I(\psi, \alpha) = \alpha(1 - q)(\psi - \theta_L)$  if it is pooling with informed governance, and  $\Delta_G(\psi) = -q(\theta_H - \psi) + (1 - q)(\psi - \theta_L)$  if it is pooling with sledgehammer governance. It is immediate that the maximal payoff improvement occurs if a separating equilibrium is played in the continuation game at time 2.

Now,  $\Delta_S(\psi) = \kappa$  when  $\psi = \psi_1$ , and  $\Delta_S(\psi) < \kappa$  for  $\psi < \psi_1$ . Hence, even if a separating equilibrium is played in the continuation game, the outsider is better off staying out than intervening when  $\psi < \psi_1$ . Since the payoff in any other equilibrium is lower, the outsider stays out for all values of  $\beta$  when  $\psi < \psi_1$ .

Next, suppose  $\psi \in [\psi_1, \psi_2]$ . We first show that  $\phi(\psi) = \frac{1}{1 + \frac{1-2\delta_L}{\theta_H - \theta_L} + \frac{(1-q)(\psi - \theta_L) - \kappa}{q\lambda_H(\theta_H - \theta_L)}}$  is strictly decreasing in  $\psi$  when  $\psi \geq \psi_1$ . Consider the denominator of  $\phi(\psi)$ . Since  $\delta_L$  is decreasing in  $\psi$ , it follows that  $\frac{1-2\delta_L}{\theta_H - \theta_L}$  is increasing in  $\psi$ . Denote the third term in the denominator as  $Z = \frac{(1-q)(\psi - \theta_L) - \kappa}{q(\theta_H - \theta_L)\lambda_H}$ . Then,  $\frac{\partial Z}{\partial \psi} = \frac{(1-q)(\psi - \theta_L) - \kappa}{q\lambda_H(\theta_H - \theta_L)} = \frac{\lambda_H(1-q) - [(1-q)(\psi - \theta_L) - \kappa](1-2\theta_H)}{q(\theta_H - \theta_L)\lambda_H^2}$ . The denominator is clearly positive. Consider the numerator:  $\lambda_H(1 - q) > 0$ , and  $1 - 2\theta_H < 0$ . Further, recall that  $\psi_1 = \theta_L + \frac{\kappa}{1-q}$ . Hence, if  $\psi \geq \psi_1$ , it follows that  $(1 - q)(\psi - \theta_L) - \kappa \geq 0$ . Therefore, the numerator of  $\frac{\partial Z}{\partial \psi}$  is strictly positive, and hence  $Z$  is strictly increasing in  $\psi$ . Hence, the denominator of  $\phi(\psi)$  is strictly increasing in  $\psi$  whenever  $\psi \geq \psi_1$ . It follows that  $\phi(\psi)$  is strictly decreasing in  $\psi$  over the same range.

Now, observe that  $\phi(\psi_1) = \frac{1}{1 - \frac{1-2\delta_L}{\theta_H - \theta_L}} = \beta_s(\psi_1)$ . By inspection,  $\phi(\psi) < \beta_s(\psi)$  when  $\psi >$

$\psi_1$ . Recall that  $\beta_\ell(\alpha, \psi) = \frac{1}{1 + \frac{1-2\delta_L}{\theta_H - \theta_L} + \frac{(1-\alpha)(1-q)\lambda_L(1-2\delta_L)}{q\lambda_H(\theta_H - \theta_L)}}$ . Now,  $\lambda_L(1-2\delta_L) = \psi - \theta_L$ , so that we can write  $\beta_\ell(\alpha, \psi) = \frac{1}{1 + \frac{1-2\delta_L}{\theta_H - \theta_L} + \frac{(1-\alpha)(1-q)(\psi - \theta_L)}{q\lambda_H(\theta_H - \theta_L)}}$ . Therefore, the condition  $\phi(\psi) > \beta_\ell(\alpha, \psi)$  is equivalent to  $(1-q)(\psi - \theta_L) - \kappa > (1-\alpha)(1-q)(\psi - \theta_L)$ , or  $\psi < \theta_L + \frac{\kappa}{\alpha(1-q)} = \psi_2$ . Also, it follows that  $\phi(\psi_2) = \beta_\ell(\alpha, \psi_2)$ .

Finally, the condition  $\kappa < \frac{\alpha q(1-q)(\theta_H - \theta_L)}{q + (1-q)(1-\alpha)}$  is equivalent to  $\psi_2 < \psi_f(\alpha)$ . Further, it is straightforward to show that  $\psi < \psi_f(\alpha)$  in turn implies that  $\beta_\ell(\alpha, \psi) > \beta_b(\psi)$ .

Therefore, for  $\psi \in (\psi_1, \psi_2)$ ,  $\phi(\psi)$  lies between  $\beta_S(\psi)$  and  $\max\{\beta_\ell(\alpha, \psi), \beta_b(\psi)\}$ . It follows from Proposition 3 part (i) that for any  $\psi$  in this range, if  $\beta = \phi(\psi)$ , a hybrid equilibrium is played in the continuation game.

Consider the payoff improvement the outsider can expect from this hybrid equilibrium. We have  $\Delta_Y(\psi, \alpha, \sigma_L) = (1-q)[1 - \sigma_L(1-\alpha)](\psi - \theta_L)$ , where  $\sigma_L$  is as defined in equation (26). When  $\beta = \phi(\psi)$ ,  $\frac{1-\beta}{\beta} = \frac{1-2\delta_L}{\theta_H - \theta_L} + \frac{(1-q)(\psi - \theta_L) - \kappa}{q\lambda_H(\theta_H - \theta_L)}$ . Hence,

$$\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{1-2\delta_L} - 1 = \frac{(1-q)(\psi - \theta_L) - \kappa}{q\lambda_H(1-2\delta_L)}.$$

Further, note that  $\lambda_L(1-2\delta_L) = \psi - \theta_L$ . Therefore,  $\sigma_L = \frac{(1-q)(\psi - \theta_L) - \kappa}{(1-\alpha)(1-q)(\psi - \theta_L)}$ , so that  $[1 - (1-\alpha)\sigma_L] = \frac{\kappa}{(1-q)(\psi - \theta_L)}$ .

Hence,  $\Delta_Y(\psi, \alpha, \sigma_L) = (1-q)(\psi - \theta_L)[1 - (1-\alpha)\sigma_L] = \kappa$ . That is, if  $\psi \in (\psi_1, \psi_2)$  and  $\beta = \phi(\psi)$ , the payoff improvement resulting from the outsider's intervention is exactly  $\kappa$ . Hence, the outsider is exactly indifferent between intervening and not.

Now, keeping  $\psi$  fixed, consider an increase in  $\beta$ . From equation (26), by inspection  $\sigma_L$  declines in  $\beta$ . Hence,  $\Delta_Y(\psi, \alpha, \sigma_L)$  increases as  $\beta$  increases. Therefore, for any  $\beta > \phi(\psi)$ , if a hybrid equilibrium is played in the continuation game at time 2, the outsider strictly prefers to enter. If  $\sigma_L$  declines to zero, a separating equilibrium is played in the continuation game. Since  $\Delta_S(\psi) > \Delta_Y(\psi, \alpha, \sigma_L)$  whenever  $\sigma_L > 0$ , the outsider again strictly prefers to enter.

Finally, consider  $\psi > \psi_2(\alpha)$ . The condition  $\kappa < \frac{\alpha q(1-q)(\theta_H - \theta_L)}{q + (1-q)(1-\alpha)}$  is equivalent to  $\psi_2 < \psi_f(\alpha)$ . Suppose first that  $\psi \in (\psi_2, \theta_f]$ . Then, it is possible that, if  $\beta$  is sufficiently low, a pooling equilibrium with informed governance is played in the continuation game at time 2. The payoff improvement if the outsider intervenes is then  $\Delta_I(\psi, \alpha) = \alpha(1-q)(\psi - \theta_L)$ . If  $\theta_{out} > \psi_2 = \theta_L + \frac{\kappa}{\alpha(1-q)}$ , it follows that  $\Delta_I(\psi, \alpha) > \kappa$ , and the outsider strictly prefers to enter. Suppose, instead, that a hybrid equilibrium is played in the continuation game. Recall that  $\beta_L(\alpha, \psi) > \phi(\psi)$  for  $\psi > \psi_2$ . As shown above, for any fixed  $\psi$ , if  $\beta > \phi$  and a hybrid equilibrium is played, the outsider strictly prefers to enter. Finally, it follows that if a separating equilibrium is played, since  $\theta > \psi_1$ , the outsider strictly prefers to enter.

Next, suppose that  $\psi > \psi_f$ . If  $\beta$  is sufficiently low, a pooling equilibrium with sledgehammer governance is played in the continuation game at time 2. The payoff improvement

if the outsider intervenes is then  $\Delta_G(\psi) = -q(\theta_H - \psi) + (1 - q)(\psi - \theta_L)$ . Evaluating this expression at  $\psi = \psi_f$ , we have  $\Delta_G(\psi_f) = \frac{\alpha q(1-q)(\theta_H - \theta_L)}{q + (1-\alpha)(1-q)} > \kappa$ . Since  $\Delta_G(\psi)$  is strictly increasing in  $\psi$ , the outsider strictly prefers to enter at all  $\psi > \psi_f$ .

On the other hand, if  $\beta$  is high enough that a hybrid equilibrium results, since  $\beta_b(\psi) > \beta_L(\alpha, \psi) > \phi(\psi)$ , the outsider again strictly prefers to enter. Finally, it follows that if a separating equilibrium is played, since  $\theta > \psi_1$ , the outsider strictly prefers to enter. ■

### Proof of Proposition 6

In a hybrid equilibrium,  $\gamma = 0$ . Hence, the board's payoff in the hybrid equilibrium may be written as

$$\Pi(\alpha) = F - c(\alpha) = q\theta_H + (1 - q)\psi - (1 - q)(1 - \alpha)\sigma_L(\psi - \theta_L) - c(\alpha). \quad (32)$$

From the expression for  $\sigma_L$  in equation (26), it follows that the term  $(1 - \alpha)\sigma_L$  is a constant that does not depend on  $\alpha$ . It is immediate that the derivative with respect to  $\alpha$  is  $\Pi'(\alpha) = -c'(\alpha) < 0$ . ■

### Proof of Lemma 5

In a pooling equilibrium with informed governance,  $\sigma_H = \sigma_L = 1$  and  $\gamma = 0$ . Substituting these into the expression for  $F$  in Lemma 4, the payoff of the board in this equilibrium may be written as

$$\Pi(\alpha) = q\theta_H + (1 - q)\psi - (1 - q)(1 - \alpha)(\psi - \theta_L) - c(\alpha). \quad (33)$$

The first-order condition with respect to  $\alpha$  is

$$c'(\alpha) = (1 - q)(\psi - \theta_L), \quad (34)$$

and since  $c(\cdot)$  is convex, the second-order condition is satisfied. Hence, if the board anticipates a pooling equilibrium with informed governance at time 2, it should set  $\alpha = \alpha_c$ . Its expected payoff is then

$$\Pi(\alpha_c) = q\theta_H + (1 - q)\psi - (1 - q)(1 - \alpha_c)(\psi - \theta_L) - c(\alpha_c). \quad (35)$$

Now, suppose the board anticipates sledgehammer governance at time 2. It should optimally set  $\alpha = 0$ . Its expected payoff is then

$$\tilde{\Pi}(0) = q\psi + (1 - q)\psi = \psi. \quad (36)$$

Comparing the two payoffs, the board prefers to set  $\alpha = \alpha_c$  and conduct informed gover-

nance when  $\theta < \psi_f(\alpha)$ . ■

### Proof of Lemma 6

In a pooling equilibrium with informed governance,  $\sigma_L = 1$  and  $\gamma = 0$ . Thus, the cash flow improvement following the outsider's intervention is  $(1 - q)\alpha(\psi - \theta_L)$ . For the outsider to intervene, this expression must be weakly greater than  $\kappa$ ; i.e.,  $\alpha \geq \frac{\kappa}{(1-q)(\psi - \theta_L)} = \alpha_e$ . ■

### Proof of Proposition 7

We begin this proof with three preliminary steps. First, we provide expressions for the the board's payoff as a function of its choice of signal  $\alpha$  and the precision of the outsider's signal  $\psi$  in each of the possible equilibria that can play out at time 2. We then demonstrate the relationship among different threshold values of  $\psi$ . And finally, we show how  $\beta_m$  changes as  $\psi$  crosses different threshold values.

#### *Preliminary step 1. Board payoff expressions*

For a fixed value of  $\alpha$ , the payoff to the board in each of the different possible equilibria is denoted as follows: In a no-governance equilibrium, it earns a payoff  $F_N(\alpha) = q\theta_H + (1 - q)\theta_L - c(\alpha)$ , in a separating equilibrium it earns  $F_S(\alpha, \psi) = q\theta_H + (1 - q)\psi - c(\alpha)$ , in a pooling equilibrium with informed governance it earns  $F_I(\alpha, \psi) = q\theta_H + (1 - q)\theta_L + (1 - q)\alpha(\psi - \theta_L) - c(\alpha)$ , in a pooling equilibrium with sledgehammer governance it earns  $F_G(\alpha, \psi) = \psi - c(\alpha)$ , and in a hybrid equilibrium it earns  $F_Y(\alpha, \psi) = q\theta_H + (1 - q) \left[ \psi - \frac{q\lambda_H}{(1-q)\lambda_L} \left\{ \frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{1-2\delta_L} - 1 \right\} (\psi - \theta_L) \right] - c(\alpha)$ . Observe that in every equilibrium in which the outsider enters, the payoff is strictly increasing in  $\psi$ .

#### *Preliminary step 2. Threshold $\psi$ values*

Define  $\psi_c$  as the solution to  $\psi_c = \theta_L + \frac{\kappa}{(1-q)\alpha_c(\psi_c)}$ . This represents the value of  $\psi$  at which the board's preferred  $\alpha$  in a pooling equilibrium with informed governance is just large enough that the it induces the outsider to enter (i.e.,  $\alpha_e(\psi_c) = \alpha_c(\psi_c)$ ). Also, define  $\psi_d$  as the solution to  $\psi_d = \theta_L + \frac{\kappa + c(\alpha_c(\psi_d))}{(1-q)\alpha_c(\psi_d)}$ . This represents the value of  $\psi$  at which  $\phi(\psi) = \beta_c(\psi)$ . We show that the following ordering holds:  $\psi_1 < \psi_3 < \psi_c < \psi_d < \psi_g$ .

$\psi_1 < \psi_3$ . Recall that  $\psi_1 = \theta_L + \frac{\kappa}{1-q}$  and  $\psi_3 = \theta_L + \frac{\kappa}{(1-q)c^{-1}(\kappa)}$ . We have  $c^{-1}(\kappa) < 1$  since  $\kappa < \infty$ , so  $\psi_1 < \psi_3$ .

$\psi_3 < \psi_c$ . We first show that  $\alpha_e(\psi_3) > \alpha_c(\psi_3)$ . We have  $\frac{\partial F_I(\alpha_e(\psi_3), \psi_3)}{\partial \alpha} = (1 - q)(\psi_3 - \theta_L) - c'(\alpha_e(\psi_3))$ . Now observe that  $\alpha_e(\psi_3) = \frac{\kappa}{(1-q)(\psi_3 - \theta_L)} = c^{-1}(\kappa)$ , using the definition of  $\psi_3$ . This implies that  $\kappa = c(\alpha_e(\psi_3))$ . Substituting this back into the expression for  $\alpha_e$  and rearranging yields  $(1 - q)(\psi_3 - \theta_L) = \frac{c(\alpha_e(\psi_3))}{\alpha_e(\psi_3)}$ . Therefore,  $\frac{\partial F_I(\alpha_e(\psi_3), \psi_3)}{\partial \alpha} = \frac{1}{\alpha_e(\psi_3)} [c(\alpha_e(\psi_3)) - \alpha_e(\psi_3)c'(\alpha_e(\psi_3))]$ . Since  $c(\cdot)$  is convex, the expression inside the brackets is negative for  $\alpha_e > 0$ , which holds since  $\alpha_e(\psi_3) = c^{-1}(\kappa) > 0$ . Therefore,  $\frac{\partial F_I(\alpha_e(\psi_3), \psi_3)}{\partial \alpha} < 0$ . Since, by definition,  $\alpha_c(\psi) = \arg \max_{\alpha} F_I(\alpha, \psi)$ , and  $F_I(\alpha, \psi)$  is strictly concave in  $\alpha$ , this implies

that  $\alpha_e(\psi_3) > \alpha_c(\psi_3)$ . Now, since  $\alpha_e(\psi_c) = \alpha_c(\psi_c)$ ,  $\alpha'_e(\psi) < 0$ , and  $\alpha'_c(\psi) > 0$ , it must be that  $\psi_3 < \psi_c$ .

$\psi_c < \psi_d$ . This follows directly from the fact that  $(\alpha_c(\psi_d)) > 0$ .

$\psi_d < \psi_g$ . Define  $\Gamma(\psi) = (1-q)(\psi - \theta_L)\alpha_c(\psi) - c(\alpha_c(\psi))$ . We can rewrite the expression for  $\psi_d$  as  $\Gamma(\psi_d) = \kappa$ , and the expression for  $\psi_g$  as  $\Gamma(\psi_g) = \psi_g - q\theta_H - (1-q)\theta_L$ . Since  $\kappa < \psi_g - q\theta_H - (1-q)\theta_L$  (from  $\kappa < \kappa_2$ ), we must have  $\Gamma(\psi_d) < \Gamma(\psi_g)$ . Observe that  $\Gamma'(\psi) = (1-q)\alpha_c(\psi) + (1-q)(\psi - \theta_L)\alpha'_c(\psi) - c'(\alpha_c(\psi))\alpha'_c(\psi)$ . Since  $c'(\alpha_c(\psi)) = (1-q)(\psi - \theta_L)$  from (9), we have  $\Gamma'(\psi) = (1-q)\alpha_c(\psi)$ , which is clearly positive. Thus we must have  $\psi_d < \psi_g$ .

*Preliminary step 3. Relationship between  $\beta_m$  and  $\psi$*

Observe that  $\psi = \psi_d$  can be rewritten as  $\phi(\psi) = \beta_c(\psi)$  and that  $\psi = \psi_g$  can be rewritten as  $\beta_c(\psi) = \beta_b(\psi)$ . It is straightforward to show that  $\phi'(\psi) < \beta'_c(\psi) < \beta'_b(\psi)$ . Therefore, we have

$$\beta_m = \begin{cases} \phi(\psi) & \text{if } \psi \leq \psi_d \\ \beta_c(\psi) & \text{if } \psi \in (\psi_d, \psi_g) \\ \beta_b(\psi) & \text{if } \psi \geq \psi_g, \end{cases} \quad (37)$$

Having established these preliminary results, we now prove each part of the proposition in turn.

*Proof of part (i)*

Suppose either that  $\psi < \psi_1$  or that  $\psi \in (\psi_1, \psi_3)$ ,  $\beta < \phi(\psi)$ . We consider each of these two cases in turn.

Case (a) From Proposition 5, if  $\psi < \psi_1$ , the activist stays out, regardless of the value of  $\alpha$  chosen by the board. If the activist stays out, governance has no value for the board ( $F'_N(\alpha) < 0$ ), and it is optimal to set  $\alpha = 0$ . Hence, there is no governance in this region.

Case (b) Suppose  $\psi \in (\psi_1, \psi_3)$  and  $\beta < \phi(\psi)$ .

Suppose the board chooses an  $\alpha$  such that the outsider enters. Since  $\phi(\psi) < \beta_s(\psi)$  for each value of  $\psi$ , the equilibrium in the continuation game cannot exhibit separation; instead, either a hybrid or pooling equilibrium must obtain. As shown in the proof of Proposition 5, if a hybrid equilibrium results in the continuation game and  $\beta < \phi(\psi)$ , the outsider will not enter regardless of the value of  $\alpha$ .

The only other possibility in which the outsider may enter is that there is a pooling equilibrium in the continuation game. We first show that the pooling

equilibrium must exhibit informed governance, and then argue that the board is better off with no governance.

Step 1 Any pooling equilibrium must exhibit informed governance.

Consider the equation that defines  $\kappa_1$ ,  $\kappa = c \left( \frac{\kappa}{q(1-q)(\theta_H - \theta_L)} \right)$ . The left-hand side is linear in  $\kappa$ , and the right-hand side is strictly convex. Hence, if  $\kappa < \kappa_1$ , it follows that  $\kappa > c \left( \frac{\kappa}{q(1-q)(\theta_H - \theta_L)} \right)$ , or  $q(\theta_H - \theta_L) > \frac{\kappa}{(1-q)c^{-1}(\kappa)}$ . Adding  $\theta_L$  to both sides, we have  $\psi_f(0) > \psi_3$ . From Proposition 4, we know that a pooling equilibrium with sledgehammer governance exists only if  $\psi \geq \psi_f(\alpha)$ . However,  $\psi_f(\alpha)$  is strictly increasing in  $\alpha$ . Hence,  $\psi_3 < \psi_f(\alpha)$  for any  $\alpha \geq 0$ , and for  $\psi < \psi_3$ , any pooling equilibrium must exhibit informed governance.

Step 2 The board prefers no governance.

In the no-governance outcome, the board optimally chooses  $\alpha = 0$  (since the outsider does not enter, it cannot be beneficial to increase  $\alpha$  beyond zero). Suppose instead the board chooses some  $\alpha > 0$ . The difference in payoffs between a pooling equilibrium with informed governance and no-governance outcome is  $F_I(\alpha, \psi) - F_N(0) = \alpha(1-q)(\psi - \theta_L) - c(\alpha)$ . Now, evaluate this expression at  $\alpha = \alpha_e(\psi) = \frac{\kappa}{(1-q)(\psi - \theta_L)}$ , which yields  $F_I(\alpha_e(\psi), \psi) - F_N(0) = \kappa - c \left( \frac{\kappa}{(1-q)(\psi - \theta_L)} \right)$ . Evaluating this last expression at  $\psi = \psi_3 = \frac{\kappa}{(1-q)c^{-1}(\kappa)}$ , we have  $F_I(\alpha_e(\psi_3), \psi_3) - F_N(0) = 0$ . That is, at  $\psi = \psi_3$ , the board is exactly indifferent between a pooling equilibrium with  $\alpha = \alpha_e(\psi)$  and a no-governance outcome with  $\alpha = 0$ .

By inspection,  $F_I(\alpha_e(\psi), \psi) - F_N(0)$  is strictly increasing in  $\psi$ , so for any  $\psi < \psi_3$ , it follows that  $F_I(\alpha_e(\psi), \psi) < F_N(0)$ . That is, the board strictly prefers no governance to a pooling equilibrium with  $\alpha = \alpha_e$ .

Now, by Lemma 6,  $\alpha_e(\psi)$  is the minimum screening level that will induce the outsider to enter. Thus, we only need to show that the board prefers no governance to a pooling equilibrium at any  $\alpha > \alpha_e(\psi)$ . But we have already seen that  $\frac{\partial F_I(\alpha_e(\psi_3), \psi_3)}{\partial \alpha} < 0$ . Therefore,  $F_I(\alpha, \psi_3) - F_N(0) = 0$  for any  $\alpha > \alpha_e(\psi)$ . So the board prefers the no-governance outcome to any pooling equilibrium with informed governance in which  $\alpha \geq \alpha_e(\psi)$ . The board then chooses  $\alpha = 0$ ; since this is clearly less than  $\alpha_e(\psi)$ , the outsider stays out.

*Proof of part (ii)*

Next, we consider part (ii) of the proposition. Suppose that  $\psi > \psi_1$  and  $\beta > \phi(\psi)$ . There are two cases to consider:  $\beta > \beta_s(\psi)$  and  $\beta \in (\beta_m(\psi), \beta_s(\psi))$ . We consider these case in turn.

Case (a) Suppose that  $\beta > \beta_s(\psi)$ . Since neither  $\psi_1$  nor  $\beta_s(\psi)$  depends on  $\alpha$ , we have  $\psi \geq \psi_1$  and  $\beta > \beta_s(\psi)$  for any  $\alpha$ . Therefore, the separating equilibrium holds regardless of  $\alpha$ . Since  $F'_S(\alpha) = -c'(\alpha) < 0$ , the board optimally chooses  $\alpha = 0$ .

Case (b) Now suppose that  $\beta \in (\beta_m(\psi), \beta_s(\psi))$ . We proceed in four steps. In the first step, we show that the hybrid equilibrium obtains in the continuation game when  $\alpha = 0$ . We next show that the board optimally chooses  $\alpha = 0$  when it anticipates a hybrid equilibrium in the continuation game. Then we show that the only other possible equilibrium is a pooling equilibrium with informed governance, which can obtain when  $\alpha$  is sufficiently large. And finally, we show that the board chooses  $\alpha = 0$  and a hybrid equilibrium over a larger  $\alpha$  and a pooling equilibrium.

Step 1 If  $\alpha = 0$ , a hybrid equilibrium obtains in the continuation game.

It is sufficient to show that the parameter space defined by  $\psi > \psi_1$  and  $\beta \in (\beta_m(\psi), \beta_s(\psi))$  lies within the parameter space in which a hybrid equilibrium obtains when  $\alpha = 0$ . Note that,  $\alpha_c c'(\alpha_c(\psi)) > c(\alpha_c(\psi))$  because  $c(\cdot)$  is convex. Substituting for  $c'(\alpha_c(\psi))$  using (9), we have  $\alpha_c(\psi)(1 - q)(\psi - \theta_L) > c(\alpha_c(\psi))$ . Inspection shows that this implies  $\beta_\ell(0, \psi) < \beta_c(\psi)$ . Since  $\beta_c(\psi) \leq \beta_m(\psi)$  by the definition of  $\beta_m(\psi)$ , we have  $\beta_\ell(0, \psi) < \beta_m(\psi)$ . Thus  $\beta \in (\beta_\ell(0, \psi), \beta_s(\psi))$ . Since  $\psi > \psi_1$  and  $\beta > \phi(\psi)$ , the outsider enters by Proposition 5, and a hybrid equilibrium obtains in the continuation game by Proposition 3.

Step 2 If it anticipates a hybrid equilibrium, the board optimally chooses  $\alpha = 0$ .

We have  $F_Y(\alpha, \psi) = -c'(\alpha) < 0$ . Therefore, if the board anticipates that a hybrid equilibrium obtains in the continuation game, it chooses  $\alpha = 0$ .

Step 3 The only possible equilibrium in the continuation game other than the hybrid equilibrium is a pooling equilibrium with informed governance, which obtains when  $\alpha$  is sufficiently large.

First, observe that  $\psi > \psi_1$  and  $\beta > \phi(\psi)$  imply that the outsider will enter by Proposition 5, so an equilibrium with no governance cannot obtain. Next, note that  $\beta > \beta_m(\psi)$  implies that  $\beta > \beta_b(\psi)$  by the definition of  $\beta_m(\psi)$ . Thus, by Proposition 4 and Lemma 3, a pooling equilibrium with sledgehammer governance cannot obtain. Finally, since  $\beta_s(\psi)$  is independent of  $\alpha$ , a separating equilibrium cannot obtain. This leaves only a pooling equilibrium with informed governance as an alternative to a hybrid equilibrium in the continuation game.

Now, observe that  $\frac{\partial \beta_\ell(\alpha, \psi)}{\partial \alpha} > 0$  and  $\lim_{\alpha \rightarrow 1} \beta_\ell(\alpha, \psi) = \beta_s(\psi)$ . So, for  $\alpha$  sufficiently large, we have  $\beta < \beta_\ell(\alpha, \psi)$ . Thus, by Proposition 2, a pooling equilibrium with informed governance can obtain in the continuation game for  $\alpha$  sufficiently large.

Step 4 The board optimally chooses  $\alpha = 0$  and implements a hybrid equilibrium rather than choosing  $\alpha$  high enough to implement a pooling equilibrium with informed governance.

Note that  $\beta > \beta_m(\psi)$  implies  $\beta > \beta_c(\psi)$  by the definition of  $\beta_m(\psi)$ . It can easily be shown that  $\beta > \beta_c(\psi)$  is equivalent to  $F_Y(0, \psi) > F_I(\alpha_c(\psi), \psi)$ . Since  $\alpha_c(\psi) = \arg \max_{\alpha} F_I(\alpha, \psi)$ , it follows that  $F_Y(0, \psi) > F_I(\alpha, \psi)$  for any  $\alpha$ . Since the hybrid equilibrium and a pooling equilibrium with informed governance are the only two possible equilibria of the continuation game, the board optimally chooses  $\alpha = 0$  and implements a hybrid equilibrium in the continuation game.

*Proof of part (iii) (a)*

We consider the case  $\psi \in (\psi_3, \psi_c)$ , in which the board chooses  $\alpha = \alpha_e(\psi)$ , and the case  $\psi \in (\psi_c, \psi_g)$ , in which the board chooses  $\alpha = \alpha_c(\psi)$ , separately.

Case (a) Suppose that  $\psi \in (\psi_3, \psi_c)$  and  $\beta < \beta_m(\psi)$ . We first show that any  $\alpha \geq \alpha_e(\psi)$  results in a pooling equilibrium with informed governance. However, choosing  $\alpha = \alpha_e(\psi)$  dominates choosing  $\alpha > \alpha_e(\psi)$ , so we can rule out  $\alpha > \alpha_e(\psi)$ . We then show that any  $\alpha < \alpha_e(\psi)$  results in the outsider not entering. Finally we show that the board prefers to choose  $\alpha_e(\psi)$  and a pooling equilibrium with informed governance over  $\alpha = 0$  and the outsider not entering.

Consider  $\alpha \geq \alpha_e(\psi)$ . One can easily show that  $\alpha \geq \alpha_e(\psi)$  is equivalent to  $\psi \geq \psi_2(\alpha)$ . We can also rewrite  $\alpha \geq \alpha_e(\psi)$  as  $(1-\alpha)(1-q)(\psi-\theta_L) \leq (1-q)(\psi-\theta_L)-\kappa$ , which implies that  $\phi(\psi) \leq \beta_\ell(\alpha_e(\psi), \psi)$ . Since  $\psi_c < \psi_f(0)$  and  $\psi'_f(\alpha) > 0$ , we have  $\psi < \psi_f(\alpha)$  for any  $\alpha$ . Therefore, by Proposition 2,  $\alpha \geq \alpha_e(\psi)$  results in a pooling equilibrium with informed governance. Since  $\alpha_e(\psi) > \alpha_c(\psi)$  if  $\psi \in (\psi_3, \psi_c)$ , we have  $\frac{\partial F_Y(\alpha_e(\psi), \psi)}{\partial \alpha} < 0$ , and so we can rule out the board choosing  $\alpha > \alpha_e(\psi)$ .

Consider  $\alpha < \alpha_e(\psi)$ . One can show that this is equivalent to  $\psi < \psi_2(\alpha)$ . Since  $\beta_m(\psi) = \phi(\psi)$  for  $\psi \in (\psi_3, \psi_c)$ ,  $\beta < \beta_m(\psi)$  implies  $\beta < \phi(\psi)$ . By Proposition 5,  $\psi < \psi_2(\alpha)$  and  $\beta < \phi(\psi)$  result in the activist staying out. If the activist stays out, it is optimal for the board to choose  $\alpha = 0$  since  $F'_N(\alpha) = -c'(\alpha) < 0$ . However,  $\psi > \psi_3$  can be rewritten as  $F_I(\alpha_e(\psi_3), \psi_3) > F_N(0)$ . So the board does not choose  $\alpha < \alpha_e(\psi)$ . Therefore, the board optimally chooses  $\alpha = \alpha_e(\psi)$ , which results in a pooling equilibrium with informed governance in the continuation game.

Case (b) We next consider the case in which  $\psi \in (\psi_c, \psi_g)$  and  $\beta < \beta_m(\psi)$ . We proceed in five steps. First, we show that  $\alpha \geq \alpha_c(\psi)$  results in a pooling equilibrium with informed governance. Second, we show that the board will choose  $\alpha = \alpha_c(\psi)$  if it anticipates a pooling equilibrium with informed governance. The final three steps

involve showing that, while lower values of  $\alpha$  will result in other types of equilibria in the continuation game, the board prefers to choose  $\alpha = \alpha_c(\psi)$  and implement a pooling equilibrium with informed governance.

Step 1 If the board chooses  $\alpha \geq \alpha_c(\psi)$ , the equilibrium of the continuation game is a pooling equilibrium with informed governance.

First, note that  $\psi > \psi_c$  can be rewritten as  $\psi > \theta_L + \frac{\kappa}{(1-q)\alpha_c(\psi)}$ . Therefore  $\psi > \psi_c$  implies that  $\psi > \psi_2(\alpha_c(\psi))$ . Since  $\psi'_2(\alpha) < 0$ , we also have  $\psi > \psi_2(\alpha_c(\psi))$  for any  $\alpha > \alpha_c(\psi)$ . Next note that  $\psi_g < \psi_f(\alpha_c(\psi))$  since  $c(\alpha_c(\psi)) > 0$ , so  $\psi < \psi_g$  implies  $\psi < \psi_f(\alpha_c(\psi))$ . Since  $\psi'_f(\alpha) > 0$ , we also have  $\psi < \psi_f(\alpha)$  for  $\alpha > \alpha_c(\psi)$ . Now observe that  $\psi > \psi_c$  can be rewritten as  $\alpha_c(\psi)(1-q)(\psi - \theta_L) \geq \kappa$ , which is equivalent to  $\beta_\ell(\alpha_c(\psi), \psi) > \phi(\psi)$ . Since  $\beta_m(\psi) = \max\{\phi(\psi), \beta_c(\psi)\}$  for  $\psi \in (\psi_c, \psi_g)$ ,  $\beta < \beta_m(\psi)$  implies that  $\beta < \beta_\ell(\alpha_c(\psi), \psi)$  in this range. Since  $\frac{\partial \beta_\ell}{\partial \alpha} > 0$ , we have  $\beta < \beta_\ell(\alpha, \psi)$  for  $\alpha > \alpha_c(\psi)$  as well. In sum,  $\alpha \geq \alpha_c(\psi)$  results in  $\psi \in (\psi_2(\alpha(\psi)), \psi_f(\alpha(\psi)))$  and  $\beta < \beta_\ell(\alpha, \psi)$ . So, by Proposition 2,  $\alpha \geq \alpha_c(\psi)$  results in a pooling equilibrium with informed governance.

Step 2 If the board anticipates a pooling equilibrium with informed governance in the continuation game, it chooses  $\alpha = \alpha_c(\psi)$ .

This follows directly from the fact that  $\alpha_c(\psi) = \arg \max_\alpha F_I(\alpha, \psi)$ . This rules out the possibility that the board optimally chooses  $\alpha > \alpha_c(\psi)$ .

Step 3 A sufficiently low  $\alpha$  can result in the outsider staying out, but the board prefers  $\alpha = \alpha_c(\psi)$  and a pooling equilibrium with informed governance.

Since  $\psi'_2(\alpha) < 0$ , a sufficiently low  $\alpha$  could result in  $\psi < \psi_2(\alpha)$  and the outsider staying out. In this case, the board optimally chooses  $\alpha = 0$  since  $F_N(\alpha) = -c'(\alpha) < 0$ . To see that the board prefers  $\alpha = \alpha_c(\psi)$  and a pooling equilibrium with informed governance over  $\alpha = 0$  and having the activist stay out, we write  $F_I(\alpha_c(\psi), \psi) > F_I(\alpha_e(\psi), \psi) > F_I(\alpha_e(\psi_3), \psi) > F_I(\alpha_e(\psi_3), \psi_3) = F_N(0)$ , where the first inequality follows from the fact that  $\alpha_c(\psi) = \arg \max_\alpha F_I(\alpha, \psi)$ , the second from the fact that  $\partial F_I / \partial \alpha < 0$  for  $\alpha > \alpha_c(\psi)$  if  $\psi > \psi_c$ ,  $\alpha_e(\psi) > \alpha_c(\psi)$ , and  $\alpha'_e(\psi) < 0$ , and the third from the fact that  $\partial F_I / \partial \psi > 0$ . The final equality can be shown by simply rewriting  $\psi = \psi_3$ .

Step 4 A sufficiently low  $\alpha$  can result in a pooling equilibrium with sledgehammer governance, but the board prefers  $\alpha = \alpha_c(\psi)$  and a pooling equilibrium with informed governance.

Since  $\psi'_f(\alpha) > 0$ , a sufficiently low  $\alpha$  could result in  $\psi > \psi_f(\alpha)$  and a pooling equilibrium with informed governance. In this case, the board optimally chooses  $\alpha = 0$  since  $\partial F_G / \partial \alpha = -c'(\alpha) < 0$ . To see that the board prefers

$\alpha = \alpha_c(\psi)$  and a pooling equilibrium with informed governance over  $\alpha = 0$  and a pooling equilibrium with sledgehammer governance, observe that we can rewrite  $\psi < \psi_g$  as  $F_G(0, \psi) < F_I(\alpha_c(\psi), \psi)$ .

Step 5 A sufficiently low  $\alpha$  can result in a hybrid equilibrium, but the board prefers  $\alpha = \alpha_c(\psi)$  and a pooling equilibrium with informed governance.

Since  $\frac{\partial \beta_\ell}{\partial \alpha} > 0$ ,  $\beta > \beta_\ell(\alpha, \psi)$  is possible for sufficiently low  $\alpha$ . This would result in a hybrid equilibrium. In this case, the board would optimally choose  $\alpha = 0$  since  $\partial F_Y / \partial \alpha = -c'(\alpha) < 0$ . For  $\psi < \psi_g$ , we have  $\beta_m(\psi) = \max\{\phi(\psi), \beta_c(\psi)\}$ . Suppose that  $\beta_m(\psi) = \beta_c(\psi)$ . It can easily be shown that  $\beta < \beta_c(\psi)$  is equivalent to  $F_Y(0, \psi) < F_I(\alpha_c(\psi), \psi)$ . Therefore, the board would choose  $\alpha = \alpha_c(\psi)$  and a pooling equilibrium over  $\alpha = 0$  and a hybrid equilibrium.

Now suppose instead that  $\beta_m(\psi) = \phi(\psi)$ . In this case  $\beta < \beta_m(\psi)$  does not rule out the possibility that  $\beta > \beta_c(\psi)$ . If  $\beta > \beta_c(\psi)$ , then we would have  $F_Y(0, \psi) > F_I(\alpha_c(\psi), \psi)$ . However, suppose that  $\alpha$  is low enough that  $\beta > \beta_\ell(\alpha, \psi)$ . Then we must have  $\beta_\ell(\alpha, \psi) < \phi(\psi)$ . This implies that  $(1 - q)(\psi - \theta_L) - \kappa < (1 - \alpha)(1 - q)(\psi - \theta_L)$ , which can be rewritten as  $\psi < \psi_2(\alpha)$ . Since  $\beta < \phi(\psi)$ ,  $\psi < \psi_2(\alpha)$  implies that the outsider stays out by Proposition 5. So, a hybrid equilibrium cannot result if  $\beta_m(\psi) = \phi(\psi)$  and  $\beta < \beta_m(\psi)$ .

In summary, when  $\psi \in (\psi_c, \psi_g)$  and  $\beta < \beta_m(\psi)$ ,  $\alpha = \alpha_c(\psi)$  results in a pooling equilibrium, the board's optimal choice of  $\alpha$  in a pooling equilibrium with informed governance is  $\alpha = \alpha_c(\psi)$ , and the board prefers this outcome to any other achievable outcome. Therefore, the board chooses  $\alpha = \alpha_c(\psi)$  and implements a pooling equilibrium with informed governance.

*Proof of part (iii) (b)*

Suppose that  $\psi > \psi_g$  and  $\beta < \beta_b(\psi)$ . We proceed to show that the board chooses  $\alpha = 0$  and a pooling equilibrium with sledgehammer governance obtains in four steps. First, we show that  $\alpha = 0$  results in a pooling equilibrium with sledgehammer governance. Next we show that if the board anticipates a pooling equilibrium with sledgehammer governance, it will choose  $\alpha = 0$ . Then we show that the only other possible equilibrium is a pooling equilibrium with informed governance, which obtains for large values of  $\alpha$ . And finally we show that the board chooses  $\alpha = 0$  and a pooling equilibrium with sledgehammer governance over  $\alpha > 0$  and a pooling equilibrium with informed governance.

Step 1 If  $\alpha = 0$ , a pooling equilibrium with sledgehammer governance obtains in the continuation game.

Note that  $\psi_g > \psi_f(0)$ , so  $\psi > \psi_g$  implies  $\psi > \psi_f(0)$ . Since  $\beta_m(\psi) = \beta_b(\psi)$  for  $\psi > \psi_g$ ,  $\beta < \beta_m(\psi)$  implies  $\beta < \beta_b(\psi)$ . So, by Proposition 4,  $\alpha = 0$  results in a pooling equilibrium with sledgehammer governance.

Step 2 If it anticipates a pooling equilibrium with sledgehammer governance, the board optimally chooses  $\alpha = 0$ .

We have  $\partial F_G / \partial \alpha = -c'(\alpha) < 0$ . Therefore, if the board anticipates that a pooling equilibrium with sledgehammer governance obtains in the continuation game, it chooses  $\alpha = 0$ .

Step 3 The only possible equilibrium in the continuation game other than the pooling equilibrium with sledgehammer governance is a pooling equilibrium with informed governance, which obtains when  $\alpha$  is sufficiently large.

First, observe that the no governance equilibrium can only obtain if  $\kappa > F_I(\alpha, \psi) - F_N(\alpha) = \psi - q\theta_H - (1-q)\theta_L$ . This contradicts the assumption that  $\kappa < \kappa_2$ . This rules out the no governance equilibrium. To see that a hybrid equilibrium cannot hold, note first that  $\beta_m(\psi) = \beta_b(\psi)$  for  $\psi > \psi_g$ . So  $\beta < \beta_m(\psi)$  implies that  $\beta < \beta_b(\psi)$ . Therefore, if  $\psi > \psi_f(\alpha)$ , a hybrid equilibrium cannot obtain by Proposition 3. Let  $\alpha_f$  denote the level of  $\alpha$  at which  $\psi = \psi_f(\alpha)$ . It can easily be shown that when  $\psi = \psi_f(\alpha)$ , we have  $\beta_\ell(\alpha, \psi) = \beta_b(\psi)$ . Therefore, when  $\alpha = \alpha_f$ , we have  $\beta < \beta_\ell(\alpha, \psi)$ . Since  $\psi'_f(\alpha) > 0$ ,  $\psi < \psi_f(\alpha)$  implies that  $\alpha > \alpha_f$ . Since  $\frac{\partial \beta_\ell}{\partial \alpha} > 0$ , we have  $\beta < \beta_\ell(\alpha, \psi)$  for any  $\alpha > \alpha_f$ . Therefore, when  $\psi < \psi_f(\alpha)$ , we have  $\beta < \beta_\ell(\alpha, \psi)$ . Thus by Proposition 3, a hybrid equilibrium cannot obtain. Since  $\beta_b(\psi) < \beta_s(\psi)$ , we can also rule out a separating equilibrium by Proposition 1.

Finally, since  $\psi'_f(\alpha) > 0$  and  $\lim_{\alpha \rightarrow 1} \beta_\ell(\alpha, \psi) = \theta_H$ , for  $\alpha$  sufficiently large, we have  $\psi < \psi_f(\alpha)$ . As we have shown above,  $\psi < \psi_f(\alpha)$  implies that  $\beta < \beta_\ell(\alpha, \psi)$  when  $\beta < \beta_b(\psi)$ . Thus, by Proposition 2, a pooling equilibrium with informed governance can obtain for  $\alpha$  sufficiently large.

Step 4 The board optimally chooses  $\alpha = 0$  and implements a pooling equilibrium with sledgehammer governance rather than choosing  $\alpha$  high enough to implement a pooling equilibrium with informed governance.

One can easily rewrite  $\psi > \psi_g$  as  $F_I(\alpha_c(\psi), \psi) < F_G(0, \psi)$ . It follows from  $\alpha_c(\psi) = \arg \max_\alpha F_I(\alpha, \psi)$  that  $F_I(\alpha, \psi) < F_G(0, \psi)$  for any  $\alpha$ . Thus the board prefers to choose  $\alpha = 0$  and implement a pooling equilibrium with sledgehammer governance rather than choosing  $\alpha$  sufficiently high to implement a pooling equilibrium with informed governance.

■

**Proof of Proposition 8**

Recall that  $\alpha_e = \frac{\kappa}{(1-q)(\psi-\theta_L)}$ . By inspection,  $\alpha_e$  decreases as  $\psi$  increases.

The value  $\alpha_c$  is defined as the value of  $\alpha$  that solves the equation  $c'(\alpha) = (1-q)(\psi-\theta_L)$ . Since  $c(\cdot)$  is convex, as  $\psi$  increases,  $c'(\alpha_c)$  must increase as well. That is,  $\alpha_c$  increases. ■

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